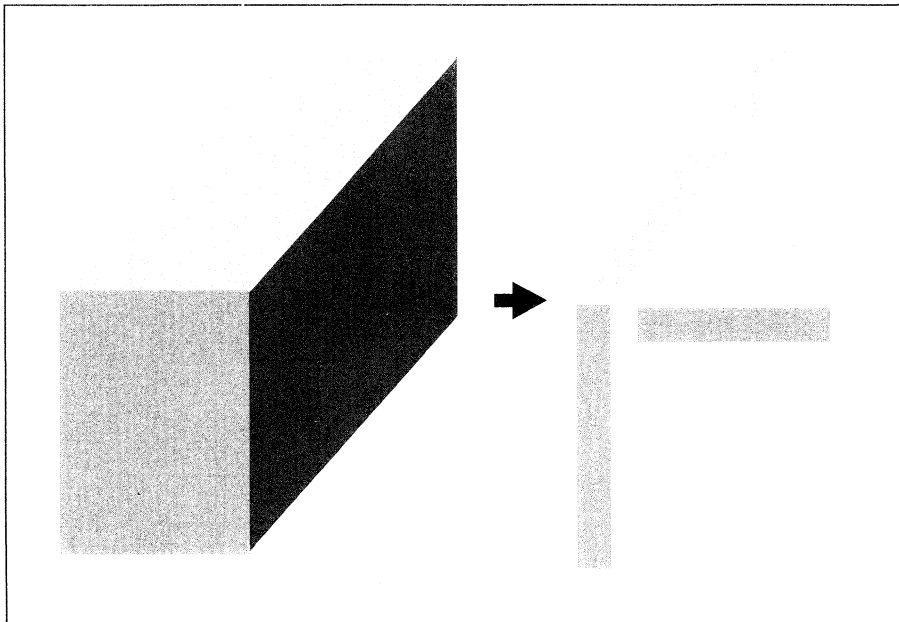


THE ANALYSIS OF THREE-WAY ARRAYS BY CONSTRAINED PARAFAC METHODS

Wim P. Krijnen



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