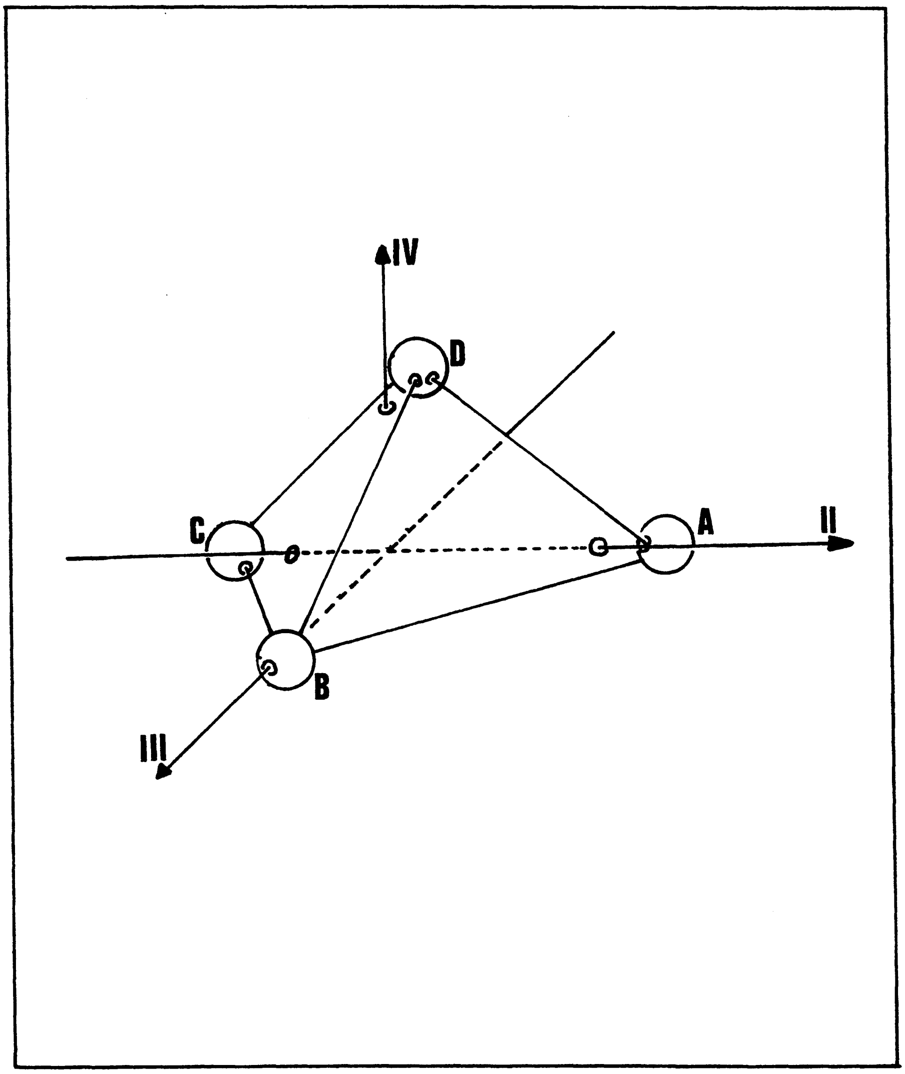
CORRELATION MATRICES

12

four ability-factor study



12.1 INTRODUCTION

In 1976, Glass issued a plea for a moratorium on data collection in favour of performing 'meta-analyses' on published research already available. His argument was that much information was available on many subjects, but that very few attempts had been made to integrate the available results. Review papers generally aim at citing research, rather than at re-analysing and integrating findings. In this chapter we will show how three-mode principal component analysis can be used for 'meta-analysis' on correlation matrices taking data from Meyers, Dingman, Orpet, Sitkei, & Watts (1964) as an illustration.

Especially in the field of intelligence tests, correlation matrices are computed for the groups used for calibrating the tests. For test developers it should be important to know whether the relationships between the subtests are the same for each calibration group. However, Wechsler (1974) for instance, published in his test manual eleven correlation matrices of subtests of the revised Wechsler Intelligence Scale for Children (WISC-R), one for each age group, without investigating the correlation matrices in a systematic fashion. In a later paper we hope to turn to an analysis of these and other correlation matrices of the WISC-R.

12.2 THREE-MODE ANALYSIS OF CORRELATION MATRICES

Using correlation matrices as direct input for a three-mode analysis introduces some complications compared to regular raw data. By taking correlation matrices, we treat the correlations as we do any raw input data, except that the input scaling poses no problems: no scaling is desirable or necessary. In fact, the data have already been scaled (jk-normalized; see section 6.5). One could treat all correlations matrices as equally important, or according to the number N of individuals in the sample. In the latter case one should multiply the correlations with $\sqrt{N}$.

Particular to the analysis of (published) correlation matrices is that one generally wants to compare the three-mode analysis with separate (two-mode) ones to assess their similarities and differences. The comparisons cannot be made directly as in three-mode analysis of correlation matrices the eigenvalues, or weights for the components, are quadratic functions of the eigenvalues from a standard principal component analysis. In other words, it is the square root of the standardized three-mode weights which should be compared with the proportion of explained variance of a regular principal component analysis.

We will only employ the Tucker2 model and not reduce the third mode, as we wish to compare the results from a three-mode solution with those of separate principal component analyses. Comparisons to assess how well the three-mode solution agrees with the separate analysis for each group can be made using the T2 core plane of that group. How this works follows from the observation that in a separate principal component analysis the correlation matrix of the k-th subgroup can approximately be decomposed as

$$R_k \cong H_k \Phi_k H_k' \qquad \text{with } \Phi_k \text{ diagonal, and } \phi_j^k \text{ the j-th eigenvalue, } j = 1,\ldots,q \text{ ,}$$

and in a three-mode analysis

$$R_k \cong H\tilde{C}_k H' \qquad \text{with } \tilde{C}_k \text{ the k-th frontal plane, which is not necessarily diagonal.}$$

Given that H and H_k are similar enough to allow comparisons, the importance of the respective axes in the two spaces can be assessed using Φ_k and $\tilde{C}_k$. It should be kept in mind that the absolute size of the entries in Φ_k and $\tilde{C}_k$ depend on the lengths of the vectors in H and H_k. In TUCKALS H is columnwise orthonormal, and in many principal component analyses H_k has columns of lengths

ϕ_i^k. In other words, the values for the loadings in a standard principal component analysis correspond to a decomposition

$$R_k = \tilde{H}_k\tilde{H}_k' \text{ with } \tilde{H}_k = H_k\,\Phi_k^{\frac{1}{2}}\ .$$

Furthermore, in the TUCKALS programs the input data are generally rescaled such that the total sum of squares is $\ell \times m \times n$. In that case some adjustment is necessary before the actual comparison can be carried out.

12.3 OTHER APPROACHES

A number of other ways exist to deal with sets of correlation matrices. Within the general framework of analysis of covariance structures, Jöreskog (1971) has developed a method which he calls *simultaneous factor analysis for several populations.* Given a theoretical model for the factor loadings the correlation matrices may be analysed jointly to see if they all fit the same hypothesized structure. For the Meyers et al. data this seems an attractive alternative to the one presented here, as a clearly defined a priori structure is available. In studies in which this is not the case, the approach seems less easy to apply.

A second way to deal with this kind of data is to perform separate component analyses for each of the correlation matrices, and compare the component loadings or weights via the *perfect congruence approach* (Ten Berge, 1977; 1982). This approach assumes, as the previous one, that a target structure is available, either from a previous study or from theoretical considerations.

Finally, the approach taken by Meyers et al. in the analysis of their data may be employed, i.e. using the same *transformational procedure* on the component solutions, and hope that they point in the same directions. Alternatively, all components may be transformed via a procrustes rotation (see, e.g. Gower, 1975) to a common target. The former approach gives little guarantee that the desired result will be obtained, the latter is shown to be suboptimal by Ten Berge (1982).

12.4 FOUR ABILITY-FACTOR STUDY: DATA, HYPOTHESES, AND ANALYSES

In their monograph Meyers et al. (1964) state that their purpose is "to explore for the presence of a factorial structure in abilities of children of preschool age", and their general hypothesis was that "at all the preschool ages investigated (i.e. 2, 4, and 6 years old), some factor differentiation has occurred" (p.7). Their way to tackle this problem was "to hypothesize four group factors and to build suitable instruments and tests for them". In addition they hypothesized increasing differentiation with increasing age, which should lead to a more detailed factorial structure at later ages, and should allow for decreasing correlations between oblique factors for the older children. Finally they put forward -tentatively- that there should be a greater factor differentiation in normal than in retarded children of the same mental age.

Table 12.1 gives an overview of tests used for the three age groups (2, 4, and 6 year olds), and detailed descriptions can be found in the original publication (p. 9-16). The normal children (85, 89, and 100 for the age groups respectively) were all "Anglo-White" Californians, and the retarded children (56, 40, and 46 for the age groups respectively) were selected from institutions primarily for their testability and for falling within desired mental age brackets (p.19). Note that the designation "two years old" for the retarded children refers only to their *mental* ages (MA), and not their *chronological* ages (CA). The chronological age for the two-year old retarded children ranged from 49 through 175 months, for the four-year olds from 81 through 211 months, and for the six-year olds from 118-214 months.

For each group the Pearson product-moment correlations of the twelve tests were determined from the raw scores (for these correlations see Meyers et al., 1964, p.24,25), the correlation matrices were subjected to a principal component analysis, and subsequently rotated with the biquartimin procedure of Carroll (1957), and with a procrustes rotation to a target loading matrix (Hurley & Cattell, 1962).

We performed a three-mode principal component analysis using the TUCKALS2 program on the six correlation matrices, which will be

Table 12.1 *Four ability-factor study: hypotheses and test names*

	Two Years		Four Years		Six Years
			Hypothesis A - Hand-Eye Psychomotor		
2-1	Bead stringing (large beads)	4-1	Bead stringing (small beads)	6-1	Bead stringing (same as 4-1)
2-2	Disk stacking (same as 4-2)	4-2	Disk stacking (same as 2-2)	6-2	PMA motor
2-3	Cube stacking (same as 4-3)	4-3	Cube stacking (same as 2-3)	6-3	Circle dotting
			Hypothesis B - Perceptual Speed		
2-4	Form-color-size matching	4-4	Pacific color-form matching	6-4	PMA picture matching
2-5	Form-color matching	4-5	Pacific figure matching	6-5	PMA figure matching
2-6	Form matching	4-6	Pacific design discrimination	6-6	Pacific form matching
			Hypothesis C - Linguistic Ability		
2-7	Pacific expressive vocabulary and expressive language check list	4-7	Pacific expressive vocabulary (objects and pictures continuous with 2-7 and 6-7)	6-7	Pacific expressive vocabulary (pictures only)
2-8	Pacific receptive vocabulary and receptive language check list	4-8	Pacific receptive vocabulary with Ammons FRPV (continuous with 2-8 6-8)	6-8	Pacific receptive vocabulary with Ammons FRPV (continuous with 4-8)
2-9	Pacific identification by-use	4-9	Response to pictures and Monroe ideational fluency	6-9	Monroe ideational fluency
			Hypothesis D - Figural Reasoning		
2-10	Pacific pattern completion	4-10	Pacific object classification	6-10	IPAT classification
2-11	Pacific form and picture comple-	4-11	Pre-Raven pattern completion	6-11	Raven matrices
2-12	Design copying (same as 4-12)	4-12	Design copying and Pacific pattern copying (continuous with 2-12 6-12)	6-12	Pacific pattern copying and design copying (continuous with 4-12)

Source: Meyers et al. (1964). p.14.

labelled N2, N4, N6, R2, R4, R6 (N = normal; R = retarded; i = age) and we also used BMDP4M (Dixon, 1981) to obtain separate principal component analyses (Method = PCA) for each of the correlation matrices. Even though Meyers et al. used a fore-runner of BMDP4M, i.e. BIMED17, we were not able to reproduce their loadings exactly; nor did a principal factor analysis (Method = PFA), be it that the latter results were somewhat closer. In the sequel we will use the PCA results from our analysis with BMDP4M rather than those of Meyers et al.

12.5 FOUR ABILITY-FACTOR STUDY: THREE-MODE ANALYSIS

Test components. In order to follow Meyers et al.'s analysis as closely as possible, four components were determined for the 12 tests, the loadings of the common space for all groups together are given in Table 12.2, and in Fig. 12.1 a visual impression is given of the spatial arrangements of the tests in the subspace spanned by the second, third, and fourth components. It is clear

Table 12.2 *Four ability-factors study: test loadings*

		components				
		general intelligence	A-C	B	D	
		1	2	3	4	
A	1	27	33	-15	-31	
Hand-Eye	2	27	44	-19	-10	
Psychomotor	3	25	45	-30	-19	
B	4	32	3	48	-10	
Perceptual	5	33	2	49	- 2	
Speed	6	32	2	43	- 4	
C	7	30	-41	-18	-18	
Linguistic	8	26	-44	-14	-29	
Ability	9	26	-34	-27	-24	
D	10	28	- 5	- 6	60	
Figural	11	30	- 9	-22	46	
Reasoning	12	30	9	-14	33	
% explained variation		48	11	8	7	74

Note: decimal points omitted

that by a non-singular transformation of the component space a set of oblique axes can be found, each of which represents one of the test groups A, B, C and D (see Table 12.1). It is equally clear that not many new insights will be gained by such a procedure.

From Table 12.1 and Fig. 12.3 it may be concluded that for two through six year old normal and retarded children a common structure of the tests is present, and that it conforms to the four (obli-

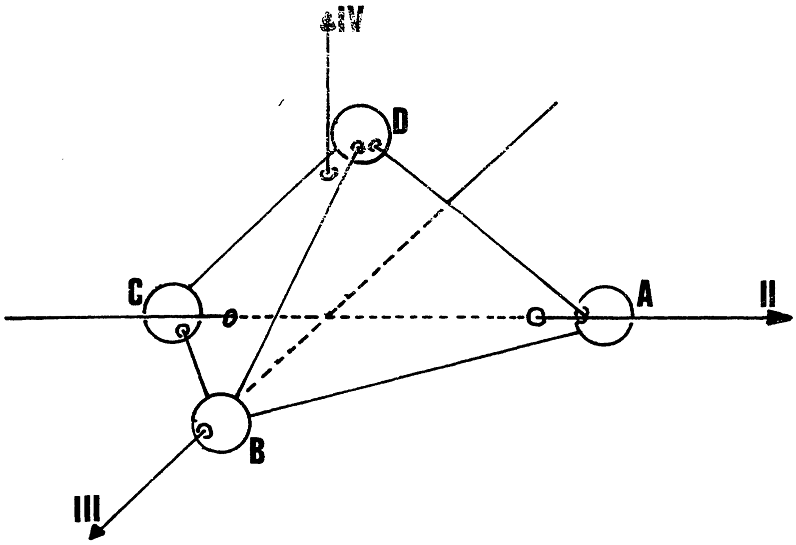


Fig. 12.1 *Four ability-factor: Spatial arrangement of test groups*

(A: Hand-eye Psychomotor; B: Perceptual Speed; C: Linguistic Ability; D: Figural Reasoning)

que) factors hypothesized by Meyers et al. It is equally acceptable to describe the structure as consisting of a general ability (intelligence) component, and a three-dimensional subspace in which the tests form a tetrahedron. The eigenvalues of the second, third, and fourth components are of the same order of magnitude, and the orientation of the axes is thus relatively arbitrary. As we shall see below, the six groups vary with respect to the order of importance of these three components.

Assessing differentiation via the core matrix. From the common component loadings no detailed statements can be made about increasing differentiation with increasing age, or the differences between retarded and normal children. How relevant the components are for each group, can be seen from the extended core matrix (Table 12.3). At this point we only discuss the diagonal elements of the core planes, and we neglect the possible interactions between the com-

Table 12.3 *Four ability-factor study: differences between groups*

A: *T2 scaled diagonal core elements* (approximate percentages explained variation)

mental age		normal 2	4	6	retarded 2	4	6	over-all
general intelligence	1	59	37	43	35	59	51	48
A vs C	2	4	11	11	14	9	14	11
B	3	8	7	7	12	6	3	8
D	4	5	6	7	6	4	8	7
sum		76	61	68	67	78	76	

B: *Separate analyses* (percentages explained variation)

mental age		normal 2	4	6	retarded 2	4	6
	1	60	38	43	35	59	51
	2	9	12	15	15	10	16
	3	6	9	10	12	7	8
	4	5	7	6	7	6	7
sum		80	66	74	69	82	82

ponents in specific groups. The off-diagonal elements are small, and never larger than the corresponding diagonal elements. The complete core matrix can, by the way, be found in section 5.5, in which these data were used to illustrate procedures for diagonalization of the extended core matrix. For comparison we have included the proportions explained variation of the separate principal component analysis for each group in Table 12.3.

The first thing to notice is that the three-mode analysis provides a fair representation for the structure in each group. The differences in amount of explained variation between the joint and the separate analysis of a group is between four to six percent. In other words, the separate analyses never succeeded in explaning more than six percent over and above the joint analysis. Even the importance attached to the various components tends to be the same in the joint analysis and the separate analyses. Note that the weights or saliences in the three-mode analysis always refer to the same axes (the rows of Table 12.3A), while in the separate analyses the axes may and do have different orientations. To illustrate the latter point the planes spanned by the second and third components for each of the separate analyses are presented in Fig. 12.2.

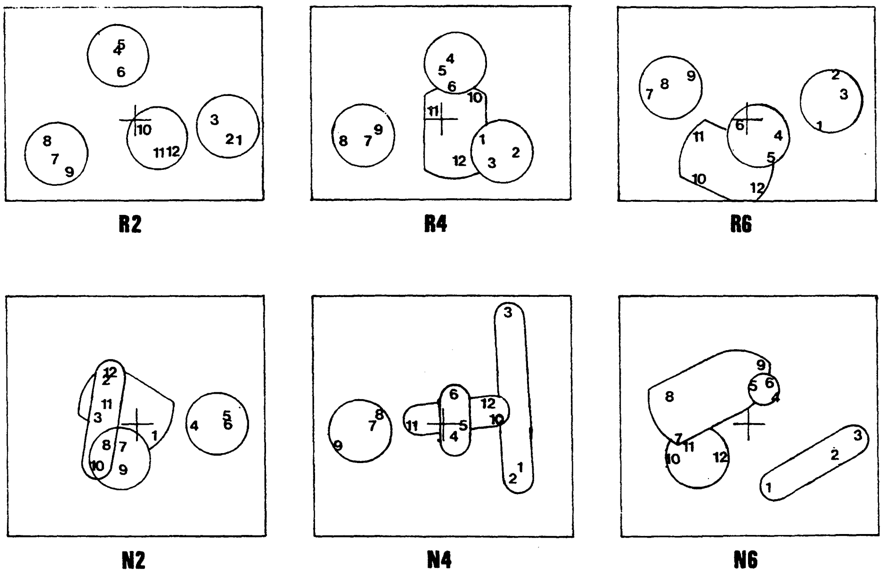


Fig. 12.2 *Four ability-factor study: Subtests spaces from separate analyses.*

(Second versus third components; 1,2,3: Hand-eye Psychomotor; 4,5,6: Perceptual Speed; 7,8,9: Linguistic Ability; 10,11,12: Figural Reasoning; details see Table 12.1)

This figure also illustrates the difficulty of comparing solutions, and of performing rotations to search for similarities between solutions.

The problem with finding suitable rotations is that, except for target rotations, there is no guarantee that even if the same structure is present this structure will actually emerge from the rotated solutions. The results from the biquartimin procedures in Meyers et al. are a case in point. Target rotations are useful in as far as one knows a priori what the structure should be, which was the case for Meyers et al. . Otherwise one could take one of the solutions as target, but then the problem arises which is the best for this purpose.

On the basis of Table 12.3 the question whether there is an increasing component differentiation can be answered. It is useful to discuss the question separately for normal and retarded children. Furthermore, as also remarked by Meyers et al., this question can only be answered within the limitations of this study, one of which is that four groups of abilities were tested, and secondly that the tests for the various ages were not the same but adapted to the specific age level. This introduces some unknowable test-age interactions.

Differentiation for normal children. Keeping this in mind, the impression is that for the *normal* children differentiation of abilities as measured by the tests occurred between ages two and four, and no further differentiation occurred between ages four and six. This conclusion is based on the 59 percent explained variation by the first component for the two-year olds, and the 37 and 46 percent for the older children. Furthermore, the distinction between linguistic abilities (C) and hand-eye psychomotor (A) on component 2 is not present for the two-year olds, but is for the older children. Note that the distinctness or coherence of perceptual speed tests (B) and figural reasoning tests (D) is the same for all age groups.

Meyers et al. did not reach the same conclusion from their analyses (p.46,47). In our opinion this is mainly due to their pre-occupation with normal-retarded comparisons at the same age levels. Furthermore they disregarded the information in the component weights or eigenvalues, and concentrated solely on loadings.

Differentiation for retarded children. The situation for *retarded* children is quite different from that of the normal ones, but one has to keep in mind that the chronological ages of the retarded children are far higher than those of the normal children. Their comparability is, in fact, an assumption by Meyers et al., which need not be true or be the same at each age level. For instance, the differentiation for the retarded two-year olds is quite as large as that for any other group, possibly suggesting that with respect to differentiation the retarded two-year olds are not comparable to normal two-year olds. On the other hand, the situation is reversed for the retarded four and six year old children. They show *less* differentiation than their normal counterparts. A possible explanation might be that differentiation is a different phenomenon for retarded than for normal children. The retarded four and six year old children confirm, by the way, Meyers et al.'s hypothesis that retarded children show less differentiation than normal children.

Again we do not reach exactly the same conclusions as Meyers et al., who state that they cannot find any differences in factor differentiation. As before they only looked at (rotated) loadings, and disregarded amounts of explained variation.

12.6 CONCLUSION

From our analyses it follows that most of the hypotheses of Meyers et al. received some (differentiation) or considerable (four factor structure) support. The difficulty for Meyers et al. was that their level of condensation was simply not high enough. For each correlation matrix with 66 data points they looked at 4 to 5 components, i.e. 48 to 60 loadings, or at 288 to 360 parameters for 396 data points. In our three-mode analysis, the information on which we based our conclusions was contained in 48 loadings and 24 core elements, or 72 parameters in all. This lack of condensation is not really Meyers et al.'s fault. The art was not as developed as it is now, and techniques for dealing with their data in a unified fashion were still being developed.

The gain of using three-mode principal component analysis in comparison with separate analyses and analytic rotation procedures can be considerable. Whether in this particular case even more insight can be obtained via a covariance structure approach is a matter for further investigation.