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Impact Scaling: Method and Application

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ABSTRACT

Impact scaling is a method of forecasting, using expert panelists' judgments of conditional and unconditional probabilities of selected possible future events, as well as the events' likely effects on such indicators as business parameters. Impact scaling to date has used the psychologically plausible INDSCAL (INdividual Differences SCALing) model of Carroll and Chang, to yield a graphic portrayal of the selected events in weighted Euclidean spaces, and to depict differences among individual panelists. Given the recent availability of a discrete psychological model (INDCLUS, for INdividual Differences CLUStering) for representing structure of the events as well as differences among panelists, we demonstrate a more elaborate and versatile set of models for implementing impact scaling. An illustration is provided using panelists of varied professional status to give judgments about impacts of possible future events on various indices of stock market performance. Finally, we consider further extensions of impact scaling to enhance the utility and precision as a model of judgment and method of forecasting.

Introduction

For deriving predictions for a panel of experts, the Delphi technique (Dalkey [14]) has seen extensive use in forecasts involving risks and a tradeoff among incompatible goals (e.g., Cicarelli [12], Nelms and Porter [34], and Preble [35]). As a practical approach for attempting to improve decision making by a committee, the Delphi technique has enjoyed considerable success. Moreover, its extensive practice has led to studies of procedures for eliciting panelists' judgments (Gustafson, Shukla, Delbecq, and Walster [25], and Press [36]) and methods of aggregating the panelists' subjective probability forecasts (Fischer [21] and Friedman [24]). However, because this procedure has no underlying model of judgment or choice behavior and yields little more than summary statistics, Delphi has not produced any new insights about the structures underlying panelists' judgments.

For representing such structures, the spatial models employed in multidimensional scaling and related techniques have proved useful for depicting perceptions of potential future events (e.g., Lundberg [30]). In advocating the use of such methods for studying the perception of risk of possible future events, Vlek and Stallen [46, p. 270] argued that " . . . multidimensional scaling analyses of data matrices containing direct personal judg-

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ments [of risk] are excellently suited for inferring relevant cognitive foundations underlying such judgments. As such, they [i.e., the analyses] provide for less explicit, less obtrusive, and therefore, perhaps more reliable ways of uncovering structures of human beliefs and values than the explicit and often prestructured questioning procedures of formal decision analysis (e.g., Keeney [26])."

Impact Scaling (Carroll [3] and Carroll and Sen [8]) seeks to combine the practical utility of the Delphi Technique with psychologically plausible models developed in multidimensional scaling. The former benefit has been heavily exploited through in-house practice of Impact Scaling (see Sen and Bozzomo [38]) by various units of the American Telephone and Telegraph Company (AT&T). However, such usage has been extremely applied and pragmatic, with little effort to interrelate underlying judgmental processes or various details of the scaling results to psychological research. In the present article, we seek to emphasize the INDSCAL model (Carroll and Chang [7]) that has been assumed in studies to date and also to introduce a more recent *discrete* model, INDCLUS (Carroll and Arabie [6]) that is potentially more appropriate to forecasting than the (continuous) INDSCAL model. Both the INDSCAL and INDCLUS models depict individual differences among the panelists from whom the data are elicited.

In evaluating the success of such a methodology, one can hardly do better than consult Keeney's [27, 28] eloquent overview of decision analysis: "Decision analysis will not solve a decision problem, nor is it intended to. Its purpose is to produce insight and promote creativity to help decision makers make better decisions. It does this by providing a methodology and procedures to decompose the problem into parts that can be meaningfully analyzed, a logic to integrate the parts, and documentation for supporting a decision to others. No analysis includes everything of importance in a decision problem. In selecting an alternative the decision makers should jointly weigh the complications of an analysis together with other factors not in the analysis. [27, p. 5] . . . Decision analysis captures the dynamic nature of decision processes. It prescribes a decision strategy that indicates what action should be chosen initially and what further actions should be selected for each subsequent event that could occur [27, p. 6]."

These quotes are reproduced here at length because they are as appropriate to Impact Scaling as to decision analysis.

Design of Study

EVENTS AND INDICES

As with the Delphi technique, a successful application of impact scaling requires a) a forecasting problem perceived as important by a group of panelists who b) qualify as experts in their familiarity with the problem. In the present study, the 14 potential future events are listed in Table 1. Selection was based on relative prominence given these and related topics by the media during the summer of 1983.

In addition to offering a graphic portrayal of the structure among these 14 events, impact scaling also seeks to represent the potential impact of these events on a series of indices or indicators gauging the performance of some typically business-related concerns. In this application, we have chosen 15 commonly cited indices, listed in Table 2, focusing on various aspects of the stock market. Selection was based largely on common inclusion in a large number of financial reports in such newspapers as the *New York Times* and the *Wall Street Journal*. As will be discussed below, a potential drawback of these particular indices is that most of our panelists perceived all pairs of the indices to be positively correlated in response to occurrence of the 14 events. (See Feeney and Hester [20] for

TABLE 1
14 Economic Events Used in Impact Scaling Study

Code Used for Plotting	Probability[a]	Economic Event
PR+	0.738	An increase of at least 1% in the prime interest rate occurs
PR−	0.302	A decrease of at least 1% in the prime interest rate occurs
TX+	0.670	There is an increase in U.S. Income Tax
TX−	0.255	There is a cut in U.S. Income Tax
IM+	0.465	The U.S. makes more money available to International Monetary Fund
IM−	0.383	The U.S. makes less money available to International Monetary Fund
O−	0.211	An oil embargo occurs (causing a shortage in the U.S.)
O+	0.331	An oil glut occurs
NUC	0.279	A breakthrough in safety and economy for nuclear electric power is achieved
TDR	0.511	There is further deregulation of the airline and the trucking industry
MMT	0.209	Much more money is made available for mass transit
OSH	0.376	OSHA guidelines are relaxed
IMQ	0.259	U.S. imposes import quotas on a large variety of manufactured products
EBT	0.404	A new electronic breakthrough, comparable to the invention of the transistor, is announced

[a]Probabilities are means from the 29 panelists of subjective unconditional probabilities.

a study of stock market indices and numerous conjectures about investors' perceptions of those indices.) In many forecasting problems amenable to impact scaling, one would expect some *negatively* correlated indices, representative of conflicting or at least somewhat incompatible indices (e.g., the cost of employee training and advancement programs versus employee turnover).

TABLE 2
Stock Market Indices Used in Impact Scaling Study and Plotting Codes for Corresponding Fitted Vectors

- Large Established Firms
  - Dow Jones Industrial Average = DJIA
  - NYSE Industrial Index = NYSEII
  - Dow Jones Transportation Average = DJTA
  - NYSE Transportation Index = NYSETI
  - Dow Jones Utilities Average = DJUA
  - NYSE Utilities Index = NYSEUI
  - NYSE Financial Index = NYSEFI
- Smaller Newer Firms
  - NASDAQ Industrial Index = NQII
  - NASDAQ Banks Index = NQBI
  - NASDAQ Insurance Index = NQINI
- Composite Parameters
  - Dow Jones Composite Average = DJCA
  - NYSE Composite Index = NYSECI
  - American Exchange Market Value Index = AEMVI
  - NASDAQ Composite Index = NQCI
  - Value Line Composite Average = VLCA
  - Subjective Unconditional Probabilities for 14 events (also referred to as scenario judged most likely) = SUCP
  - Subjective Unconditional Probabilities for Optimistic Scenario = SUCPI

The academic application described below sought to model the impact that various events and combinations of such events might have on the different indices. In a typical business application, planners would have to decide which indices were beneficial (and thus to be maximized) and which were harmful (and thus to be minimized) to the sponsoring agency/corporation. The planners would then seek to allocate available resources to influence the probabilities of possible future events so as to produce the desired mix of values on the indicators. A hypothetical example would be to lobby for a tax break to construct a new manufacturing plant (the event) with economical facilities for employee training (Index A) to reduce employee turnover (Index B).

PANELISTS

The present study used three groups of panelists: 13 MBA students, eight Ph.D. students, and eight stock brokers for a total of $m = 29$ panelists. The first two groups were all specializing in finance at the Business School of the University of Illinois at Urbana-Champaign, and the members of the third group were all employed by the Champaign office of a national firm belonging to the New York Stock Exchange. (Data from another Ph.D. student and a ninth stock broker were not used because those participants failed to follow the instructions described below.) Completing the two questionnaires required 2–2.5 hours for most subjects. Students were paid $60 and brokers $100 for their participation.

DATA COLLECTION

Impact Scaling requires three types of data: a) subjective unconditional probabilities of the occurrence of each of the events (see Table 1); b) subjective judgments (e.g., conditional probabilities) of the impact of each event on each of the other events; and c) subjective judgments of the percentage change in each index if each given event were to occur.

Elaborating on b), these data are obtained by asking each panelist to judge the (subjective) conditional probability of event E_j occurring given that event E_i has recently occurred. Note that both probabilities $P(E_j|E_k)$ and $P(E_k|E_j)$ are needed for impact scaling; in the former E_j is the impacted event and in the latter is the impacting event. The events are viewed both as having impact on each other and on business parameters as well. (See Moskowitz and Sarin [32] and Moskowitz and Wallenius [33] for a comprehensive discussion of assessing subjective conditional probabilities.)

The data were collected over two sessions per panelist. Panelists worked alone and were not permitted to confer with each other while completing the questionnaires described below. The graduate students participated during 19 April–4 May 1984 and the stock brokers during 11–18 July 1984. The questionnaire used in the first session began with a list of the 14 events. Then, after an explanation of probabilities (viz., defining the unit range and using the toss of a die as an example), the events were presented in a different random order for each panelist, and the task was to give a subjective (unconditional) estimate of the probability that each event would happen in the near future. The rest of the first session's questionnaire was devoted to obtaining the subjective conditional probabilities for all distinct pairs of events. Considering the events in the same order (unique for each subject) as for the unconditional probabilities, panelists were told to assume that the first event has indeed occurred. Then with that event serving as a "standard," subjective conditional probabilities were elicited for all remaining 13 events. On the next page, the second event served as standard for comparison with the first, third, fourth, etc. Thus, after completing the first questionnaire, each subject had given 14 unconditional probability judgments and $14 \times 13 = 182$ conditional probability judgments.

Since the judgments described in c) above require detailed familiarity with the stock market indices listed in Table 2, the authors prepared a 40-page tutorial explaining the computation of and stocks included in each index, and copies were given to the panelists to study prior to the second session. Copies of the tutorial were also available to the panelists while participating in the second session. The questionnaire for that session began with a listing of the 15 indices in a format slightly more elaborate than Table 2. Following on successive pages were the 14 events, randomly permuted for each panelist, with the request to assume that the given event had occurred. Under the event were the 15 indices, always in the order given in Table 2. Respondents were asked to estimate the magnitude of change (and the sign) of each index as a percentage if the event occurred "in the near future." For example, if the event were occurrence of an oil shortage, a respondent might indicate that the NASDAQ Industrial Index would change by −5%. In more limited and constrained practical applications, a more specific time interval (e.g., the end of the fiscal quarter) would be appropriate. At the end of the second questionnaire, each respondent had provided 15 estimates of percentage change for each of the 14 events.

Models

REVIEW OF MODELS BASIC TO IMPACT SCALING

Impact Scaling relies heavily on three models developed in the psychometric literature. Readers already familiar with these models may wish to turn to the next section.

1. Tersely stated, the INDSCAL (for INdividual Differences SCALing) method places n stimuli (events) in a Euclidean space of specified dimensionality, so that events perceived to be closely related are positioned near each other, whereas relatively unrelated events are distantly placed from each other. Differences among subjects (panelists) are depicted by stretching or shrinking (i.e., weighting) axes (dimensions) according to the salience (w_{it}, defined below) imputed to those axes by the individual panelists' data. As a result, the INDSCAL model is sometimes referred to as the "weighted Euclidean model."

Formally, the estimated distance between a pair of events E_j and E_k in the weighted Euclidean space can be written as

$$D_{jk}^{(i)} = \sqrt{\sum_{t=1}^{r} w_{it}(x_{jt} - x_{kt})^2}, \tag{1}$$

where $D_{jk}^{(i)}$ is the distance between the events E_j and E_k for the ith panelist, w_{it} is the weight for panelist i in the tth dimension, and x_{jt} is the coordinate of event E_j on the tth dimension.

Graphically, the 14 events are represented in a space of r dimensions, and the panelists are positioned in a separate space of the same dimensionality r. Two points should be mentioned about the panelists' weight space. First, the coordinates for the panelists (represented as points in the space) are given by the weights, w_{it}. Second, in impact scaling, each panelist is represented in the weight space by two points: one for judgments of impacting events and another for impacted events. Since the impact data consist of a series (viz., two from each panelist) of square $n \times n$ matrices derived from the subjective probabilities elicited from the panelists, the impact data are three-way: panelists × events × events.

A noteworthy aspect of the INDSCAL model is that, unlike two-way nonmetric multidimensional scaling in Euclidean space and most approaches to factor analysis, the axes in an INDSCAL event space have a mathematically preferred orientation. As such, the goodness-of-fit (variance accounted for; see [9], pp. 83–89) of a spatial solution is only invariant over reflections and permutations of the axes. (The data analyst, of course, is responsible for the substantive interpretation of these axes or dimensions.) The preferred orientation is conferred upon the event space by the presence of the weights, w_{it}, in eq. (1). As noted above, these weights are the coordinates of the panelists' space. Although not constrained to be positive, the weights empirically are nearly always in the positive orthant of the space. Psychologically, the weights w_{it} gauge the salience of dimension t for panelist i by differentially stretching or shrinking that dimension so that differences in projections along corresponding axes contribute more or less, respectively, to the estimated distance in eq. (1). Statistically, the size of the weights gives an indication of the variance accounted for in the data from a given panelist.

To summarize the description of the INDSCAL model and link it to the equations below, it is helpful to note the following details concerning several variables: The INDSCAL model seeks to place the events E_k in an r-dimensional Euclidean space, with the $n \times r$ matrix $\mathbf{X}$ containing the *coordinates* of each of the n events. The dimension *weights* for the panelists in a separate r-dimensional space are given by the $r \times m$ matrix $\mathbf{W}$. (In impact scaling, the dimensions of $\mathbf{W}$ are $r \times 2m$, since each panelist is represented twice.) Both of these matrices, $\mathbf{X}$ and $\mathbf{W}$, are fitted to the data by the INDSCAL program. The other variables that appear in the following discussion are derived from the panelists'data and from $\mathbf{X}$ and $\mathbf{W}$, using regression techniques and the properties of Euclidean distance.

2. The *vector* model, as used in impact scaling, assumes that a spatial solution for the events has already been obtained, and that a vector $\mathbf{p}_i$ is to be fitted, defined as a length, and a set of cosines defined by the angles between vectors and the axes of the events space. An example of such a vector would be the subjective unconditional probabilities for each of the fifteen events as judged by the ith panelist. Figure 1 shows an example of five events $A \ldots E$ (in circles) positioned in a two-dimensional event space. Two subjects' vectors have been fitted through this space. Interpreting the direction indicated by the vectors' arrows as one of greater (judged subjective) probability, one panelist has given a set of unconditional probabilities with E_c as most and E_b as least likely. The other vector depicts likelihood in reverse alphabetical order.

Note that the assumption in this artificial example is that, although the events are positioned in an INDSCAL-derived multidimensional space, probability of future occurrence can be represented as an embedded one-dimensional manifold, pointing from the most to least likely of the events. This geometric assumption is central to the use of multidimensional scaling as the basic model of forecasting in impact scaling.

As noted by Carroll [4], the vector model as fitted for ratings of subjective probabilities can be formulated algebraically as

$$\pi_{ij} \cong F_i(p_{ij}) \cong \sum_{t=1}^{r} b_{it}x_{jt} \qquad (2)$$

where π_{ij} = transformed subjective unconditional probability of event j for panelist i, p_{ij} = observed estimate of probability for E_j by the ith panelist, x_{jt} = INDSCAL coordinate of jth event on tth dimension, b_{it} = *importance* of dimension t for panelist i (it is, technically, a direction number, or proportional to the "direction cosine" of the angle

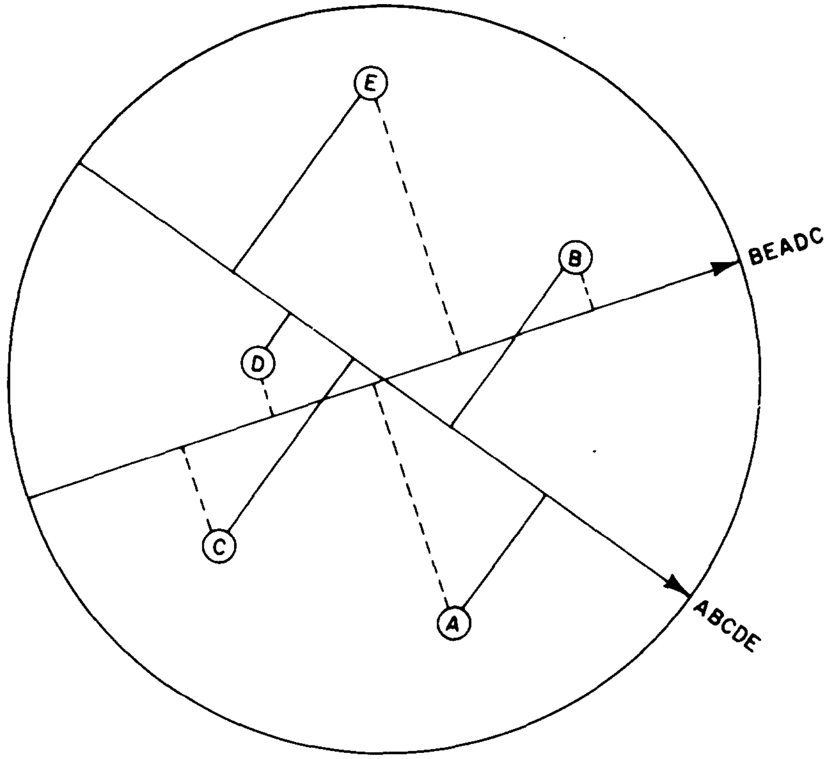

Fig. 1. Illustration of vector model, with events represented as circled letters whose projections on the vector reconstruct subjective unconditional probabilities of the corresponding events, as elicited from panelists. (Reproduced from Figure 1 of Carroll [4]).

subject i's vector makes with the dimension t coordinate axis), and r = number of dimensions. In matrix form

$$\mathbf{\Pi} \cong \mathbf{BX}, \tag{3}$$

where $\mathbf{\Pi} \equiv \{\pi_{ij}\}$ is the matrix of transformed subjective probability values π_{ij} linearly transformed by the function F_i, which takes observed probability scale values (p) into underlying values (π).

3. INDCLUS (for INdividual Differences CLUStering) describes a model and algorithm (Carroll and Arabie [5, 6]) that constitutes a *discrete* counterpart to the INDSCAL spatial model. In principle, INDCLUS is applicable to any data suitable for an INDSCAL analysis and is an individual differences generalization of the Shepard–Arabie [42] ADCLUS model for overlapping clustering and the Arabie–Carroll MAPCLUS [1] algorithm for fitting the ADCLUS model.

The crucial difference between INDSCAL and INDCLUS is that the former represents structure by a set of *dimensions* that are common to all panelists, whereas the latter uses (discrete) *clusters* or subsets of events (for which overlap is allowed—in keeping with the fact that INDCLUS is a generalization of ADCLUS/MAPCLUS) common to all panelists. These clusters are often interpreted as "features" (cf. Tversky [45]) of the set of events (or other stimuli). The dimensions (INDSCAL) or clusters (INDCLUS) are useful aids to planners in the crucial problem of determining relevant variables in forecasting (Einhorn and Hogarth [17]). In both models, individual differences among panelists are represented by weights fitted to individual panelists' data, in order to gauge salience of the dimensions (INDSCAL) or of the clusters (INDCLUS) in the solution (cf. Dietz, Fogler, and Smith [16]).

The INDCLUS model is described in terms of *similarities* as input data. Use of impact scaling (described above) to date has employed *dissimilarities* data, as discussed below in detail. Since INDCLUS, like INDSCAL, is a linear model assuming interval scale data, the dissimilarities for impacting and impacted events are first linearly transformed to be similarities, with two matrices per panelist, so that there are a total of $2m$ matrices, just as in the analyses using INDSCAL. If we let s^i_{jk} be the derived similarity between events E_j and E_k for the ith similarities matrix ($i = 1, \ldots, 2m$), then the INDCLUS model states that

$$\hat{s}^i_{jk} \cong \sum_{t=1}^{r} w_{it} g_{jt} g_{kt} + u_i, \tag{4}$$

where $\hat{s}^i_{jk}$ is the predicted similarity between events E_j and E_k, for the ith similarity matrix, r is the number of clusters (specified by the data analyst) with index t, w_{it} is the weight (assumed to be nonnegative) fitted to the tth subset (cluster) of events for input matrix i, and g_{jt} is unity if E_j is present in cluster t, otherwise zero. Finally, u_i is simply an additive constant for the ith similarity matrix, corresponding to the first (unit) column vector in eq. (17). Alternatively, u_i can be viewed as the weight (not assumed to be nonnegative) of an $(r + 1)$st "universal" cluster comprising the complete set of n events. Finally, it should be noted that the weights w_{it} for the INDCLUS model in eq. (4) are unrelated to the weights for INDSCAL in eq. (1).

Verbally, the similarity between a pair of events (as derived from the data of an individual panelist) is simply the sum of the weights (fitted to that panelist's data) of the clusters that contain both those events. (The product of $g_{jt}g_{kt}$ will be zero unless both terms are unity, implying that both E_j and E_k are constituents of cluster t.) Note that all panelists are assumed to use the same set of r clusters (of the n events), but are assumed to be differentially weighting those clusters. Thus, the weights of the clusters vary both as a function of cluster and panelist.

Equation (4) can be recast in matrix notation as

$$\hat{\mathbf{S}}^{(i)} = \mathbf{G}\mathbf{W}^{(i)}\mathbf{G}' + \mathbf{U}^{(i)}, \tag{5}$$

where $\hat{\mathbf{S}}^{(i)}$ is the symmetric $n \times n$ similarities matrix estimated for the ith input matrix, $\mathbf{G}$ is the $n \times r$ *binary* matrix whose unities within a column define the constituency of the column corresponding to the tth cluster ($t = 1, \ldots, r$), $\mathbf{W}^{(i)}$ is an $r \times r$ diagonal matrix having weights for the r clusters in its principal diagonal, as fitted to the ith input matrix, $\mathbf{G}'$ is the $r \times n$ matrix transpose of $\mathbf{G}$, and $\mathbf{U}^{(i)}$ is the constant matrix supplying the additive constant required by linear regression for the ith input matrix. Thus, to the input similarities data $\mathbf{S}^{(i)}$ ($i = 1, \ldots, 2m$), INDCLUS simultaneously fits the r clusters ($\mathbf{G}$) and their weights ($\mathbf{W}^{(i)}$). If it were not for the binary constraint on $\mathbf{G}$ in eq. (4), then this model would simply be a generalization of principal components analysis, with $\mathbf{W}$ corresponding to eigenvalues and $\mathbf{G}$ to eigenvectors, and with the attendant rotational invariance leading to the "rotation problem." However, the discrete, binary constraint on $\mathbf{G}$ requires much more elaborate procedures for fitting the INDCLUS model.

One point should be made to help the reader bridge the gap from INDSCAL to INDCLUS. Specifically, in an INDSCAL analysis, the events are positioned in a weighted Euclidean space having r dimensions (where r is specified by the data analyst) with continuously valued coordinates along those dimensions. Although programs for fitting the INDSCAL model (Pruzansky [37]) generally do not constrain the axes (dimensions) to be orthogonal (i.e., uncorrelated), they usually turn out to be approximately so. An INDCLUS solution can be viewed as a discrete version of principal components, for which each axis is a cluster, and each event has a coordinate of either 1 (if a constituent member of the cluster), or 0 otherwise, for each dimension. These "cluster axes" will not be orthogonal, however, unless the clustering happens to be a partition. Also, the origin of an INDCLUS solution, interpreted in this spatial framework, is nonarbitrary, unlike the situation for models like INDSCAL that enjoy a more familiar spatial interpretation.

INDCLUS offers a more natural way of representing aggregates of events than does INDSCAL. As noted, however, INDSCAL was the most relevant approach available during the original development of impact scaling, and INDCLUS was not yet available. The conceptual and other advantages INDCLUS offers for impact scaling should greatly improve the usefulness of this approach to forecasting.

MODEL UNDERLYING IMPACT SCALING

Representing the events. As noted above, each panelist i gives a subjective unconditional probability estimate of each event E_j at time T:

$$\tilde{p}^i_{Tj} \; (j = 1, \ldots, n; \, i = 1, \ldots, m).$$

Also, each panelist i provides subjective *conditional* probabilities of event E_k occurring at time $T + 1$, given that E_j occurred at time T:

$$P(E_k \text{ at } T + 1 \mid E_j \; at \; T) = \tilde{p}^i_{Tj} \qquad (j = 1, \ldots, n; \, k = 1, \ldots, n; \, j \neq k).$$

Note that a tilde above a probability denotes that the variable (data) was elicited from a panelist. For the ith panelist, the "cross-impact" of E_j on E_k is defined as

$$\tilde{c}^i_{jk} = \tilde{p}^i_{jk} - \tilde{p}_k \quad ; \tag{6}$$

that is, the subjective conditional probability of E_k given E_j minus the subjective unconditional probability of E_k. (In addition, each panelist estimates impacts of each event on each of s external business parameters, but those data will be discussed below.)

Two methodological points should be noted. First, unpublished applications of impact scaling where all panelists participated simultaneously often used consensual procedures for eliciting unconditional probabilities. Since many of our panelists were run in separate sessions, we obtained unconditional probabilities from each panelist, and subtracted the unconditional probability specific to each panelist in eq. (6). Second, some readers may be surprised that we used the unmodified estimates of probabilities provided by the panelists, instead of attempting to "improve" (Lindley [29]) or "update" (Diaconis and Zabell [15]) their probabilities. While the future may allow adopting some of these strategies, the inherent drawback is that most of them rely on being able to state, determine, or otherwise assume a base rate for a given event. Many of the events listed in Table 1 (e.g., a new electronic breakthrough) defy such a determination, but there can be little doubt that an announcement of their occurrence can have impact on the stock market.

Returning to our explication of the model (Carroll [3] and Carroll and Sen [8]) underlying impact scaling, note that the vector of *hypothetical* unconditional probabilities has the form $\mathbf{p}_{Tj} = (p_1, p_2 \cdots 1 \cdots p_n)$ where the 1 constitutes the jth component. Impact scaling assumes the panelist is using a model of the form

$$\mathbf{p}_T = \boldsymbol{\pi}_T \mathbf{X}' + \text{error}, \tag{7}$$

where $\mathbf{p}_T$ is the unconditional probability (dependent variable), $\boldsymbol{\pi}_T$ is an r-element row vector of *relative* probabilities (that can be normalized to a total of unity across the r dimensions) unrelated to previous usage of the same variable name, and $\mathbf{X}'$ is the $r \times n$ transpose of the coordinates matrix fitted by INDSCAL in past usage. If the INDCLUS model is instead used to supply the coordinates, then in the present notation $\mathbf{X}' = \mathbf{G}'$, where $\mathbf{G}'$ is the matrix of binary (0,1) values [assumed in the INDCLUS model of eq. (4) and (5)] defining cluster membership in each of r (possibly overlapping) clusters fitted by the INDCLUS computer program.

Standard least squares regression techniques yield an estimate of the vector of relative probabilities (associated with the dimensions and later used for predicting the likelihood of various scenarios):

$$\hat{\boldsymbol{\pi}}_{Tj} = \mathbf{p}_{Tj}\, \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}. \tag{8}$$

To estimate relative probabilities at time $T + 1$, it is necessary to solve for an $r \times r$ transition matrix $\mathbf{K}$:

$$\hat{\boldsymbol{\pi}}_{(T+1)j} = \hat{\boldsymbol{\pi}}_{Tj}\mathbf{K}. \tag{9}$$

Using eq. (7), the unconditional theoretical probability of E_j at Time $T + 1$ is thus

$$\hat{\mathbf{p}}_{(T+1)j} = \hat{\boldsymbol{\pi}}_{(T+1)j}\mathbf{X}'. \tag{10}$$

Combining the results of the preceding equations, we have

$$\begin{aligned} \hat{\mathbf{p}}_{(T+1)j} &= \mathbf{p}_{Tj}\, \mathbf{X}\, (\mathbf{X}'\mathbf{X})^{-1}\, \mathbf{K}\, \mathbf{X}' \\ &= \mathbf{p}_{Tj}\mathbf{X}\, \mathbf{Z}\, \mathbf{X}', \end{aligned} \tag{11}$$

where

$$\mathbf{Z} = (\mathbf{X}'\mathbf{X})^{-1}\, \mathbf{K}. \tag{12}$$

($\mathbf{Z}$ is an $r \times r$ matrix that can be interpreted as a matrix of impacts among the r dimensions.)

Consider the situation if we had begun with the assumption that E_j did *not* occur, so that $\mathbf{p}_{T\bar{j}} = (p_1, p_2 \cdots 0 \cdots p_n)$ with a zero for the jth component.

Then we can form a vector of *cross-impacts* of E_j on events 1–n [cf. eq. (6)] as

$$\begin{aligned} \mathbf{c}_j &\equiv \mathbf{p}_{(T+1)j} - \mathbf{p}_{(T+1)\bar{j}} \\ &= (\mathbf{p}_{Tj} - \mathbf{p}_{T\bar{j}})\, \mathbf{X}\, \mathbf{Z}\, \mathbf{X}' \\ &= \boldsymbol{\varepsilon}_j\, \mathbf{X}\, \mathbf{Z}\, \mathbf{X}', \end{aligned} \tag{13}$$

where $\boldsymbol{\varepsilon}_j = (0, 0 \cdots 1 \cdots 0)$.

In matrix notation,

$$\mathbf{C} = \mathbf{I}\,\mathbf{X}\,\mathbf{Z}\,\mathbf{X}' = \mathbf{X}\,\mathbf{Z}\,\mathbf{X}', \tag{14}$$

where $\mathbf{C}$, a panelist's cross-impact matrix, has the vectors $\mathbf{c}_j$ ($j = 1, \ldots, n$) of eq. (13) for its rows. $\mathbf{I}$ is the identity matrix, which happens to be the matrix whose rows are the row vectors $\boldsymbol{\varepsilon}_j$ defined above.

For estimating $\mathbf{X}$ and $\mathbf{K}_j$ recall that the cross-impacts (data) $\tilde{c}^i_{jk}$ were defined in eq. (6). To use them as input to either Pruzansky's [37] SINDSCAL program for fitting INDSCAL (Carroll and Chang [7]) model, or INDCLUS (Carroll and Arabie [5, 6]), some preprocessing is required to render the data as an $n \times n$ matrix of "profile" (Cronbach and Gleser [13]) or "indirect" (Shepard [41]) dissimilarities between all pairs of events E_j and E_j'. As noted earlier, we can regard each event as impacting *and* being impacted upon by the other events, so that in actuality we form *two* such matrices of indirect dissimilarities for each panelist.

Define $\mathbf{D}_i^{(\text{row})} \equiv d_{ijj'}^{(\text{row})}$ as

$$d_{ijj'}^{(\text{row})} = \left[\sum_{k=1}^{n} (\tilde{c}^i_{jk} - \tilde{c}^i_{j'k})^2\right]^{1/2}, \tag{15}$$

and define $\mathbf{D}_i^{(\text{col})} \equiv d_{ijj'}^{(\text{col})}$ by

$$d_{ijj'}^{(\text{col})} = \left[\sum_{k=1}^{n} (\tilde{c}^i_{kj} - \tilde{c}^i_{kj'})^2\right]^{1/2}. \tag{16}$$

Assume

$$\mathbf{C}_i = \mathbf{X}\,\mathbf{Z}_i\mathbf{X}', \qquad i = 1, 2, \ldots, m,$$

and that $\mathbf{X}$ is of the form

$$\mathbf{X}_{(r+1)} \equiv \begin{bmatrix} k & \cdot & \\ k & \cdot & \\ k & \cdot & \\ \cdot & \cdot & \mathbf{X}_{(r)} \\ \cdot & \cdot & \\ \cdot & \cdot & \\ k & \cdot & \end{bmatrix}, \tag{17}$$

where $1\mathbf{X}_{(r)} = 0$ (i.e., $\mathbf{X}_{(r)}$ is column-centered).

Then, if we transform $\mathbf{D}_i^{(\text{row})}$ and $\mathbf{D}_i^{(\text{col})}$ to scalar products $\mathbf{B}$ via standard (e.g., Torgerson [44, p. 258]) equations [i.e., $b_{ijj'} = -\frac{1}{2}\,(d^2_{ijj'} - d^2_{i\cdot j'} - d^2_{ij\cdot} + d^2_{i\cdot\cdot})$, where the · indicates a mean of the squared distances over the corresponding subscript(s)], it can be shown that

$$\begin{aligned} \mathbf{B}_i^{(\mathrm{row})} &= \mathbf{X}_{(r)}\mathbf{J}_{(r,r+1)}\mathbf{Z}_i\,\mathbf{X}'_{(r+1)}\mathbf{X}_{(r+1)}\mathbf{Z}'_i\,\mathbf{J}'_{(r,r+1)}\mathbf{X}'_{(r)}, \\ \mathbf{B}_i^{(\mathrm{col})} &= \mathbf{X}_{(r)}\mathbf{J}_{(r,r+1)}\mathbf{Z}'_i\,\mathbf{X}'_{(r+1)}\mathbf{X}_{(r+1)}\mathbf{Z}_i\,\mathbf{J}'_{(r,r+1)}\mathbf{X}'_{(r)}, \end{aligned} \tag{18}$$

where

$$\mathbf{J}_{(r,r+1)} \equiv \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} = \left[\begin{array}{c|c} 0' & \mathbf{I}_r \end{array}\right]. \tag{19}$$

Let

$$\begin{aligned} \mathbf{J}\mathbf{Z}_i\,\mathbf{X}'_{(r+1)}\mathbf{X}_{(r+1)}\mathbf{Z}'_i\,\mathbf{J}' &= \mathbf{W}_{i1}, \\ \mathbf{J}\mathbf{Z}'_i\mathbf{X}'_{(r+1)}\mathbf{X}_{(r+1)}\,\mathbf{Z}_i\mathbf{J}' &= \mathbf{W}_{i2}. \end{aligned} \tag{20}$$

We make the simplifying assumptions that $\mathbf{W}_{i1}$ and $\mathbf{W}_{i2}$ are diagonal for all i. Then

$$\begin{aligned} \mathbf{B}_i^{(\mathrm{row})} &= \mathbf{X}_{(r)}\mathbf{W}_{i1}\mathbf{X}'_{(r)}, \\ \mathbf{B}_i^{(\mathrm{col})} &= \mathbf{X}_{(r)}\mathbf{W}_{i2}\mathbf{X}'_{(r)}. \end{aligned} \tag{21}$$

The substantive meaning of these two simplifying assumptions is hard to state precisely, but, roughly speaking, it embodies a kind of generalized statistical independence or lack of correlation among the superevents corresponding to INDSCAL dimensions or INDCLUS clusters. That is, we are not assuming that the events are orthogonal, but that some specific but unknown linear combinations of the events are.

Let $\tilde{\mathbf{D}}$ be the $n \times n \times 2m$ array defined by combining $\mathbf{D}_i^{(\mathrm{row})}$ and $\mathbf{D}_i^{(\mathrm{col})}$ for the m subjects, so that $\tilde{\mathbf{B}}$ is the $n \times n \times 2m$ array whose two-way (events × events) slices are $\mathbf{B}_i^{(\mathrm{row})}$ and $\mathbf{B}_i^{(\mathrm{col})}$ for $i = 1,2, \ldots, m$.

Then an INDSCAL analysis of $\tilde{\mathbf{D}}$ is equivalent to a symmetric CANDECOMP (Carroll and Chang [7]) analysis of $\tilde{\mathbf{B}}$, and so provides estimates of $\mathbf{X}_{(r)}$ and of $\mathbf{W}_{i1}$ and $\mathbf{W}_{i2}$ for all i.

Given this estimate of $\mathbf{X}_{(r)}$, we may define $\mathbf{X}_{(r+1)}$ by appending an additional column defined as $k\mathbf{1}'$ ($k = 1/\sqrt{n}$) and then estimate $\mathbf{Z}_i$ for each i by least squares procedures. Given the estimate $\mathbf{Z}_i$, of this matrix, we may then estimate $\mathbf{K}_i$ by

$$\hat{\mathbf{K}}_i = (\mathbf{X}'\mathbf{X})\hat{\mathbf{Z}}_i. \tag{22}$$

In general, we are only interested in the $r \times r$ submatrix of $\mathbf{K}_i$ (or of $\mathbf{Z}_i$) that corresponds to $\mathbf{X}_{(r)}$ containing the coordinates of the events in an r-dimensional weighted Euclidean space. It will turn out, however (because of orthogonality of $\mathbf{1}'$ and $\mathbf{X}_{(r)}$), that $\hat{\mathbf{K}}_{i(r)}$ calculated from $\mathbf{X}_{(r)}$ alone will identically equal the approximate $r \times r$ submatrix of $\hat{\mathbf{K}}_{i(r+1)}$.

By way of summary, we have defined a cross-impact matrix for each of the m panelists [by subtracting the relevant subjective unconditional probabilities from the subjective conditional probabilities for each panelist, as defined in eq. (6)]. Then, *two* matrices of event dissimilarities for each panelist are defined by eqs. (15) and (16). The resulting $2m$ matrices of dissimilarities among the n events define an $n \times n \times 2m$ array suitable for an INDSCAL (or an INDCLUS) analysis. The dimensions (of the preferred orientation in the INDSCAL solution) define the matrix $\mathbf{X}_{(r)}$, which is supplemented by a constant (column) vector to define the "superevents" matrix $\mathbf{X}$. The columns of this matrix can be viewed as defining coordinates of the actual manifest events on the $(r + 1)$ underlying superevents; the $(r + 1)$st vector of constants corresponds to an average of the manifest events and is required by the mechanics of linear regression. In practice, the dimensions of this superevent space have been shown to provide useful feedback to managers and planners at AT&T (Carroll [3]). Moreover, using straightforward regression techniques, one can obtain estimates of the probabilities of occurrence of each of the superevents. The dimensions can be conceptualized as defining "areas of concern" that are continuously valued collections of actual events. Note that the probability of each manifest event can be viewed as a monotonic function of the value of the underlying superevent. Each dimension, or superevent, can be thought of as being defined by certain events that are compatible with or symptomatic of that superevent, and others that are incompatible with it (or antisymptomatic of its occurrence). These would be events that have highly positive and highly negative coordinates on the dimension, respectively. Those events with intermediate, or near-zero, coordinates could be viewed as unrelated, or irrelevant, to the superevent.

Note that, in the preceding discussion of "superevents," we are having to consider (continuous) linear combinations of (discrete) events, since INDSCAL dimensions are inherently part of a space posited to be continuous. In contrast, the clusters from an INDCLUS analysis are discrete (viz., an event either is or is not included in a cluster), and this discrete representation of events may extend the applicability and enhance the utility of impact scaling.

IMPACT OF EVENTS ON EXTERNAL PARAMETERS OR INDICATORS

In the beginning of the section on data collection, we noted that in addition to eliciting subjective conditional and unconditional probabilities for events, estimates of predicted impacts of the n events on each of s selected external business parameters (listed in Table 2) are also collected. Concretely, panelists are asked to estimate the percentage (which is not constrained to be positive) change for a given parameter (e.g., size of work force) if a specific event occurs.

Formally, let $y_{j\nu}$ be the judged impact of E_j on parameter (or indicator variable) ν. $\mathbf{y}_\nu$ is a (column) vector of impacts on variable ν. $\mathbf{Y}_i \equiv \{y_{j\nu}^{(i)}\}$ is the $n \times s$ matrix of the impacts for panelist i. Let $\mathbf{\Psi}_i$ be this matrix of hypothesized impacts of the latent superevents on parameters. We then assume the following linear model:

$$\mathbf{Y}_i = \mathbf{X}\mathbf{\Psi}_i + \text{error} \qquad (= \overline{\mathbf{Y}}_i + \text{error}), \tag{23}$$

leading to the following least squares estimate of the $r \times s$ matrix $\mathbf{\Psi}_i$ (having coordinates of the $\mathbf{y}$ vectors):

$$\hat{\boldsymbol{\Psi}}_i = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}_i, \tag{24}$$

so that

$$\hat{\mathbf{Y}}_i = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}_i. \tag{25}$$

The "subjective expected value" of (percentage change in) the parameters may be defined using the model as

$$\begin{aligned} \mathbf{e}_i &= (e_{i1}, e_{i2} \cdots e_{is}) = \mathbf{p}_i\overline{\mathbf{Y}}_i \\ &= \boldsymbol{\pi}_i\,(\mathbf{X}'\mathbf{X})\boldsymbol{\Psi}_i. \end{aligned} \tag{26}$$

Analogously, *estimated* subjectively expected values (of change) are provided by

$$\begin{aligned} \hat{\mathbf{e}}_i &= \hat{\mathbf{p}}_i\hat{\mathbf{Y}}_i = \hat{\boldsymbol{\pi}}_i(\mathbf{X}'\mathbf{X})\hat{\boldsymbol{\Psi}}_i \\ &= \hat{\mathbf{p}}_i\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}_i. \end{aligned} \tag{27}$$

In practice, the vector $\mathbf{y}_\nu$ is usually the mean percentage of change estimated by all the panelists for a given business parameter. Considering this vector of data as input, the program PROFIT (Chang and Carroll [10]) uses linear regression to fit a vector $\boldsymbol{\Psi}_i$ in the INDSCAL or INDCLUS space of events (cf. Figure 1) so that the projections of the n events on the vector optimally reconstruct the observed percentages of $\mathbf{y}_\nu$.

Thus, the INDSCAL or INDCLUS solution can be used to embed a vector for each of the 15 business parameters (viz., stock market indices) used in this study. The cosine of the angle between the vector for a given parameter and the axis of any particular dimension in a space can be viewed as gauging the impact the events associated with the dimension would have on that parameter.

As noted earlier, yet another vector to be fitted in the events space is the one based on the mean unconditional probability estimates over all the panelists. The direction of the corresponding vector fitted by PROFIT can be regarded as the (unidimensional) "scenario judged most likely" in the multidimensional event space. Of course, one of the purposes to which forecasting is often directed is to seek a strategy for influencing certain future events so as to optimize some parameter or combination of parameters. Thus, if a committee of panelists decides that resources can be directed toward changing the probability of one or more of the events, then a new $\mathbf{y}_\nu^*$ vector, corresponding to the more optimistic scenario, can be positioned by PROFIT in the event space, so as to see how the revised scenario would affect each of the business parameters. Although somewhat more tenuous, there is the possibility of a group of planners also revising their estimates of the percentage change for selected indices in response to occurrence of given events. For example, a conglomerate enterprise seeking to maximize overall profit could offset the expected effect of (for example) a change in the NASDAQ Insurance Index with a contingency plan of selling its holdings in insurance companies.

Results

SINDSCAL EVENT SPACE

Fifty-eight matrices (two from each panelist), generated according to eqs. (15) and (16), served as input to Pruzansky's [37] SINDSCAL program for fitting the INDSCAL

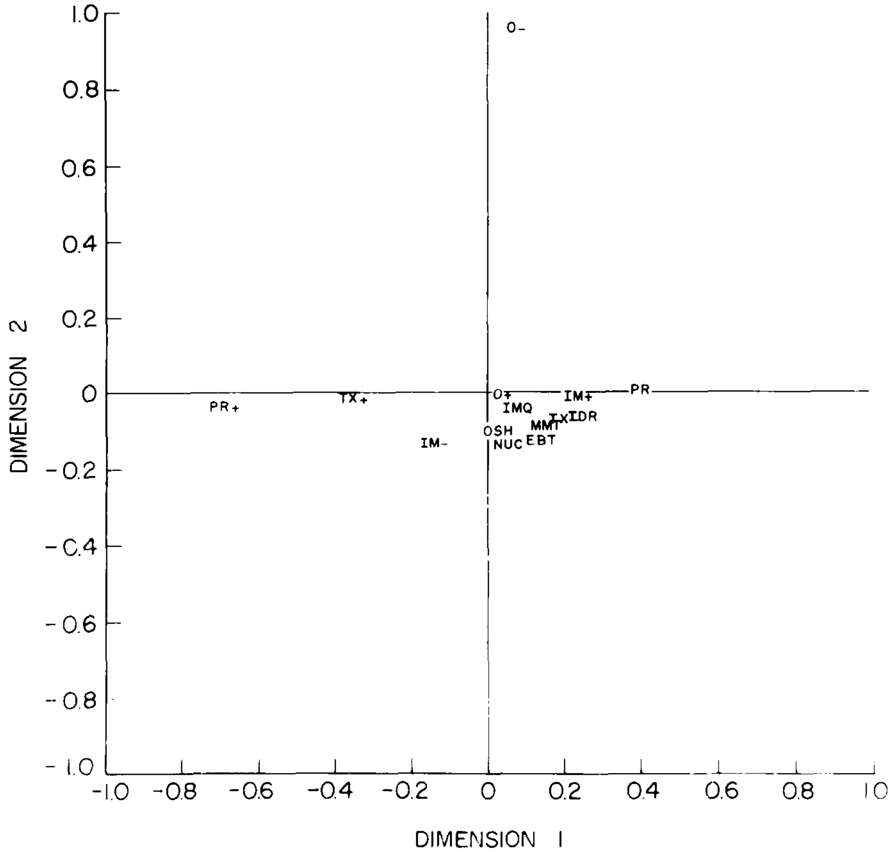

Fig. 2. Dimensions 1 and 2 of four-dimensional SINDSCAL event space. Plotting codes for events are given in Table 1.

model. Using several different initial configurations, the best fitting solutions in six through two dimensions gave variances accounted for (VAF), respectively, of 44.1, 39.7, 34.3, 28.6, and 21.8%. The four-dimensional solution was selected for giving the best tradeoff between goodness-of-fit and interpretability.

Figure 2 shows a plot of the first two dimensions of the four-dimensional event space. (The abbreviated codes used for plotting the events are given in Table 1.) The first dimension is easily interpreted as governmentally controlled and/or influenced economic policies. Specifically, the dimension is anchored by increase versus decrease of the prime rate, with tax increase and more funds for the International Monetary Fund (IMF) also prominent. The second dimension obviously reflects a concern about energy and pits an oil shortage against all the remaining 13 events. The first two dimensions accounted for approximately 12.2 and 8.4% of the variance, while the third and fourth, respectively, accounted for 7.2 and 6.5%. Figure 3 shows a plot of these dimensions. The third depicts fiscal (e.g., tax increase) versus monetary (e.g., increase in the prime rate) events. The interpretation of the fourth dimension of the INDSCAL event space is somewhat problematic. With the notable exception of changes in the oil supply, this dimension seems to contrast events associated with a stable U. S. economy (e.g., less

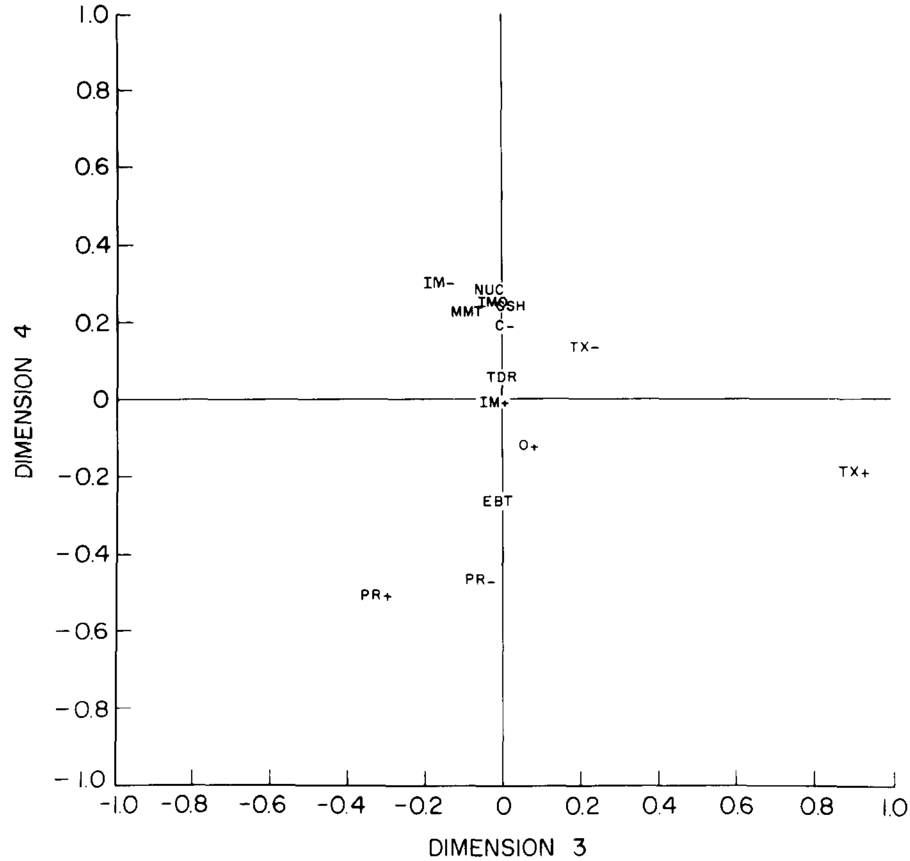

Fig. 3. Dimensions 3 and 4 of four-dimensional SINDSCAL event space. Plotting codes for events are given in Table 1.

money for IMF, enhanced safety, and efficiency for nuclear power) versus those that perturb it (e.g., changes in the prime rate).

We noted above that fitting the INDSCAL model also results in a subject or panelist space, separate from but having the same dimensionality as the event space. Figure 4 presents the panelist space corresponding to the first two dimensions of the event space shown in Figure 2. The plotting codes used in Figure 4 require some explanation. An "R" (row) as the first character denotes a panelist's judgment of the events impacting each other, whereas a "C" (column) denotes judgments of events being impacted [cf. eqs. (15) and (16) above]. The eight brokers are designated by BRKA, BRKB, etc. For student panelists, the two digits after the "C" or "R" are simply for clerical purposes. Following that number, as "M" indicates MBA, and "P" indicates Ph.D. The final digit tells the student's year of study.

The most striking aspect of Figure 4 is the differential impacted/impacting weights for the two dimensions. Concretely, the first dimension is much more salient (viz., heavily weighted) when its events are viewed as effects (impacted), whereas the second dimension is more salient when the events are judged as causes (impacting). Carroll [3, p. 514] noted that a similar finding often occurred for one or two of the dimensions in unpublished

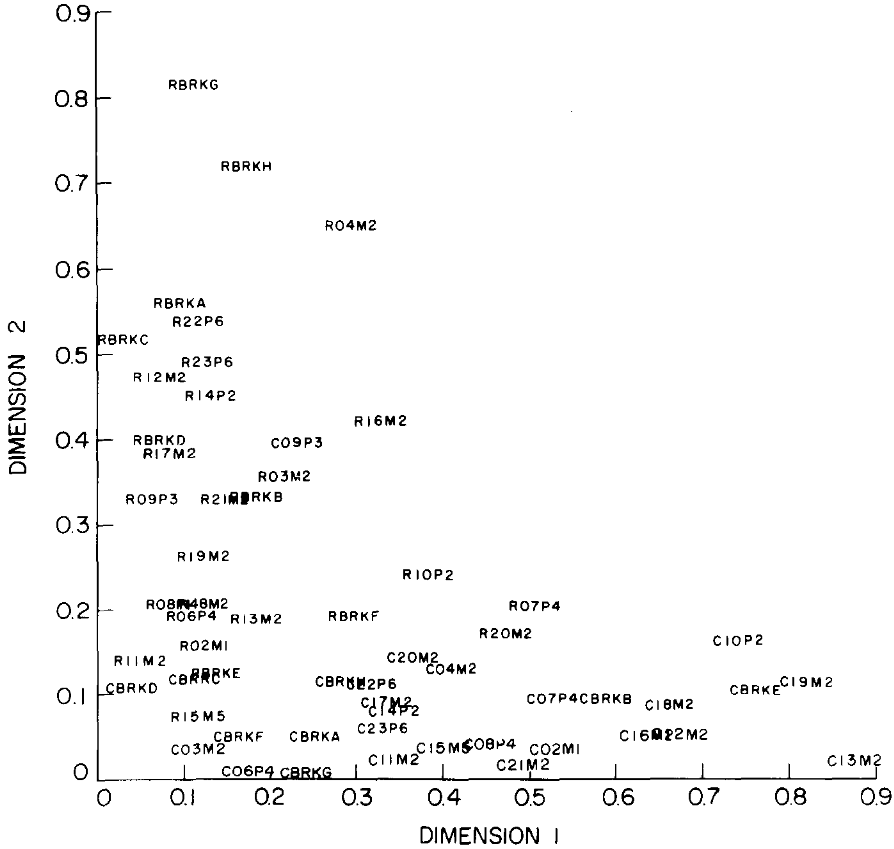

Fig. 4. Dimensions 1 and 2 of four-dimensional SINDSCAL panelist space.

studies conducted by AT&T. Although not presented here, the plot for dimensions one and three shows a similar although not quite as clear-cut pattern, with the first dimension again being more heavily weighted when viewed as impacted. In the plot (again, not presented) of dimensions one and four, the same pattern is still somewhat in evidence. Considering differences among panelists in Figure 4, it is interesting to note that the stock brokers (with the exception of points CBRKB and CBRKE) gave greater emphasis to the second dimension (energy). There is a less pronounced tendency for the Masters students to emphasize the first dimension more heavily. Weights for students in the doctoral program seemed to be evenly distributed over the two dimensions.

In Figure 5, showing dimensions three and four of the panelist space, no predisposition toward the impacting/impacted emphasis is apparent. There is, however, some evidence that the Masters students emphasize the third dimension more than they do the fourth dimension. A similar tendency is less marked for the doctoral students.

We noted earlier that impact scaling also yields spatial representations depicting the interrelationships of the events and the business parameters, with the latter represented as vectors embedded in the event space. As a practical detail, it should be added that the program PROFIT (Chang and Carroll [10]) fits the data described above on the left side of eq. (23) as vectors in the INDSCAL-supplied event space. Figure 6 presents the same

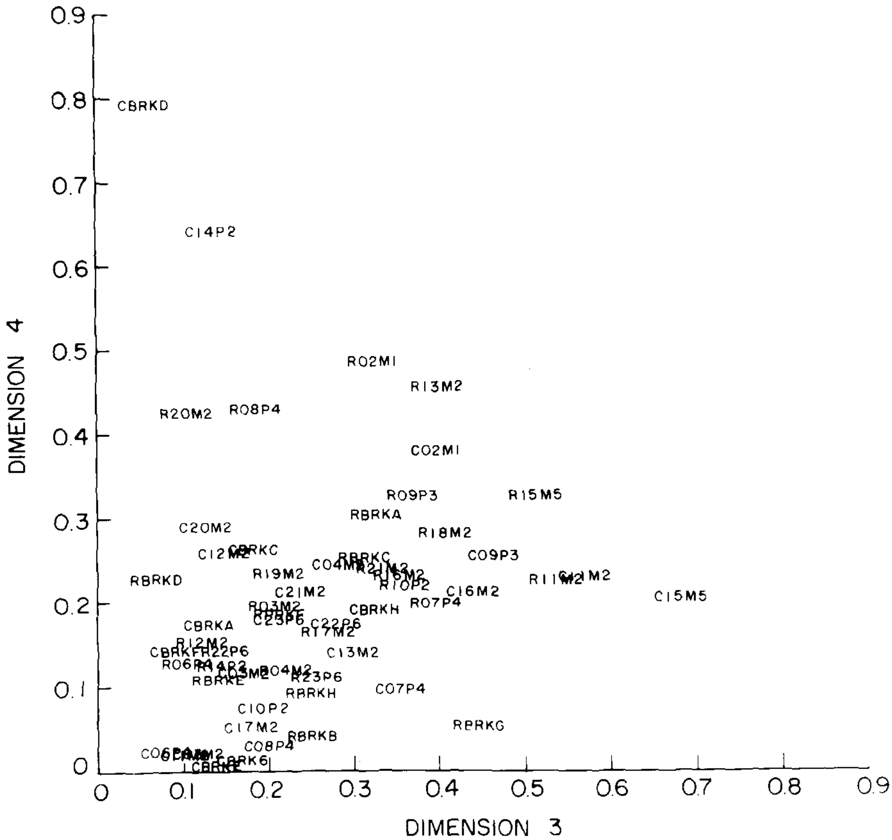

Fig. 5. Dimensions 3 and 4 of four dimensional SINDSCAL panelist space.

event space as in Figure 2, but with vectors representing the 15 stock market indices and the unconditional probabilities embedded in the space. The direction in which the vectors for all the market indices point in Figure 6 indicates that our panelists viewed an electronic breakthrough (EBT), relaxation of OSHA rules, etc., as beneficial to the stock market, in contrast to such deleterious events as an oil shortage (O−). (It might be noted that the length of each vector is proportional to the square root of its variance accounted for, also interpretable as a multiple correlation [R], and that all vectors pass through the origin because INDSCAL coordinates are centered, i.e., have a mean of zero, for each dimension.)

The most striking aspect of Figure 6 is that the scenario judged most likely (the vector for subjective unconditional probabilities, labeled SUCP) is approximately orthogonal to the vectors for the parameters (stock market indices). The implication is that the panelists' mean judgment saw the future (along these two dimensions) as neither helping nor hindering the performance of the stock market. Such a view would be consistent with the hypothesis of the efficient market (Fama [19]), contending that predictions based on past events are useless in forecasting price changes. Of course, our panelists were asked to make judgments about future events, but the panelists may have viewed the task as an extrapolation based on knowledge of the past. Note that the (unidimensional) path judged for the future is directed toward an increase in the prime lending rate, less money for the International Monetary Fund, etc. However, it should

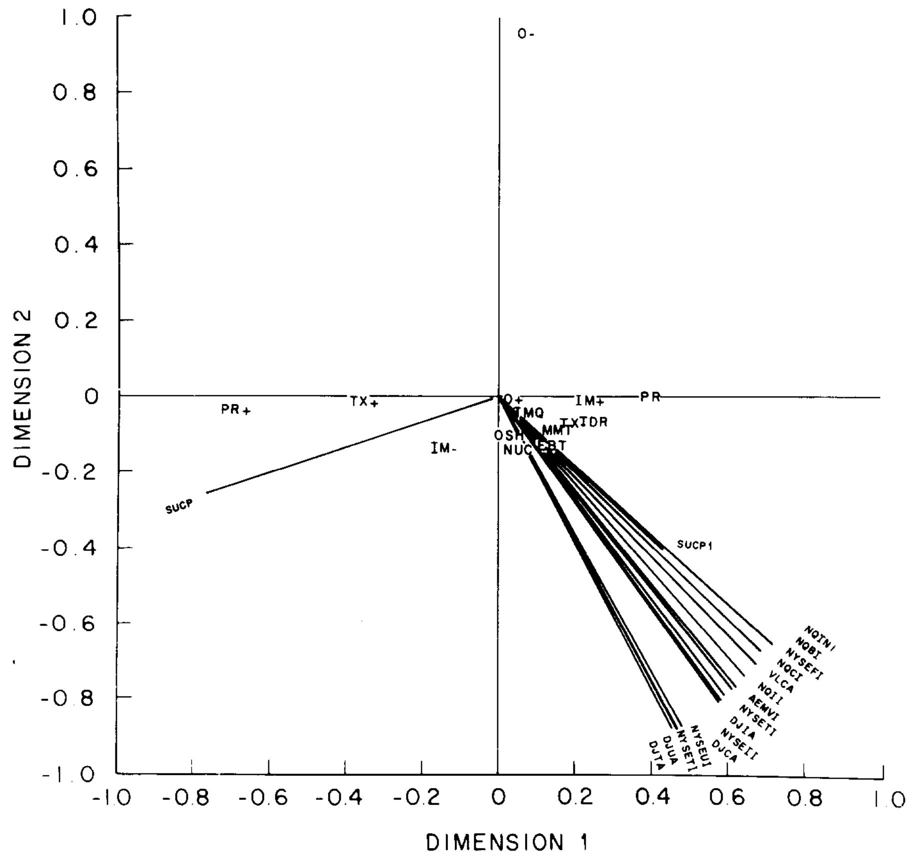

Fig. 6. Stock market indices and subjective unconditional probabilities (SUCP) represented as vectors embedded in dimensions 1 and 2 of four-dimensional SINDSCAL event space.

be noted that the events space we obtained for INDSCAL was four-dimensional, and the figures in this paper necessarily present only two-dimensional projections of the four-dimensional space. The orthogonality just noted was apparent in the two-dimensional projection shown in Figure 6, but not in the other five possible projections, one of which (Figure 7) will be presented below.

A reassuring aspect of that figure is that the business parameters are grouped together in Table 2 as "composite indices" are all positioned closely together. Most of the indices for "large established firms" fell together, but the Financial and the Industrial Indices of the NYSE were mixed with the other two types of indices, as presented in Table 2.

Figure 7 shows the same event space as Figure 3, but with the vectors now embedded. Unlike their directions in Figure 6 (and in the other four two-dimensional projections not presented), the vectors now show some divergence. Specifically, the two indices relevant to utilities (Dow Jones Utilities Average and NYSE Utilities Index) are pointing toward quadrant one, whereas the other vectors are in the direction of quadrants three and four. The scenario judged most likely (SUCP), however, would not be propitious toward these two indices.

Figures 2–7 graphically depict the panelists' perceptions of the events and their impacts upon the indices. In-house use of impact scaling at AT&T has demonstrated that planners have found the dimensional representations of events useful for organizing their

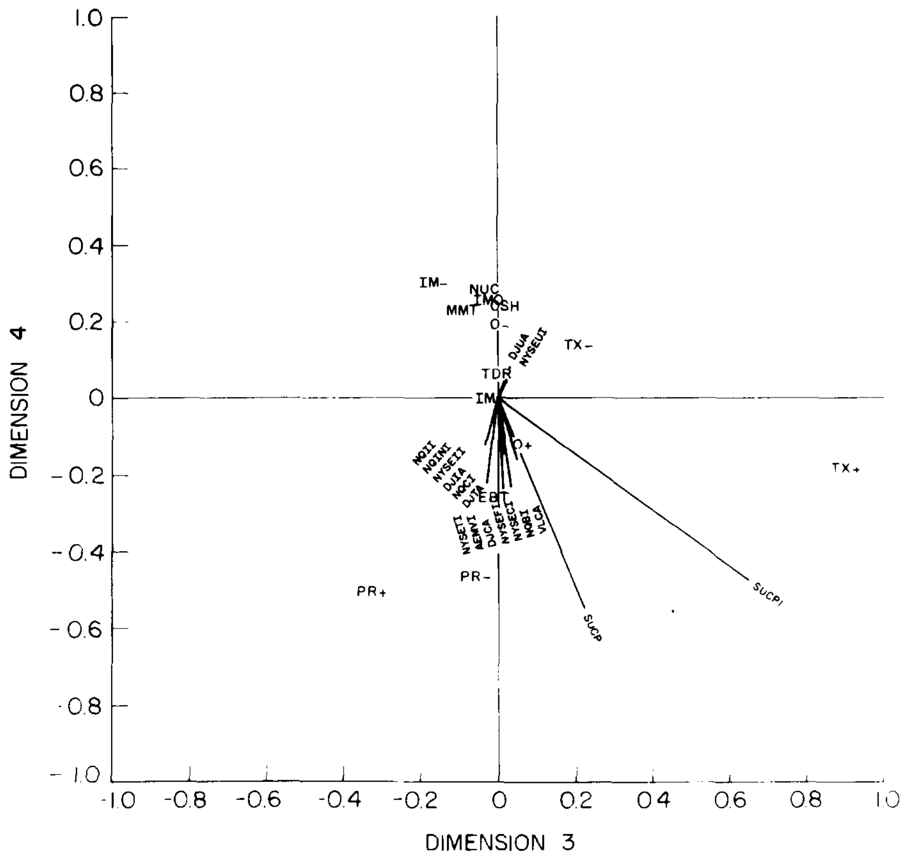

Fig. 7. Stock market indices and subjective unconditional probabilities (SUCP) represented as vectors embedded in dimensions 3 and 4 of four-dimensional SINDSCAL event space.

conceptualizations of possible scenarios of the future (Sen and Bozzomo [38]). However, we noted above in the discussions of Figures 6 and 7 that trying to visualize vectors from a four-dimensional space in two-dimensional projections of that space can be misleading. Carroll and Sen [8], therefore, devised a useful auxiliary way of presenting the vectors so as to note which events were of strategic importance to the parameter corresponding to a given vector. Concretely, those authors provided the "impact plot" as a straightforward scatterplot of projections of the events on the subjective unconditional probability vector versus projections on the vector corresponding to any given index. If an index is to be maximized in the future scenario, then such an impact plot should suggest strong positive correlation between these two sets of projections.

Figure 8 shows an "impact plot" using the NASDAQ Insurance Index. (Note that centering of the coordinates accounts for the negative coordinates or scale values.) Planners seeking to maximize this parameter would wish to increase the probability of a drop in the prime lending rate (moving its point into quadrant one) and decrease the probabilities of a tax increase and of an increase in the prime rate (moving their points from quadrant four to quadrant three). Further moving the point for an oil shortage to the left (i.e., reducing the probability of occurrence) would also facilitate a scatterplot indicative of high positive correlation.

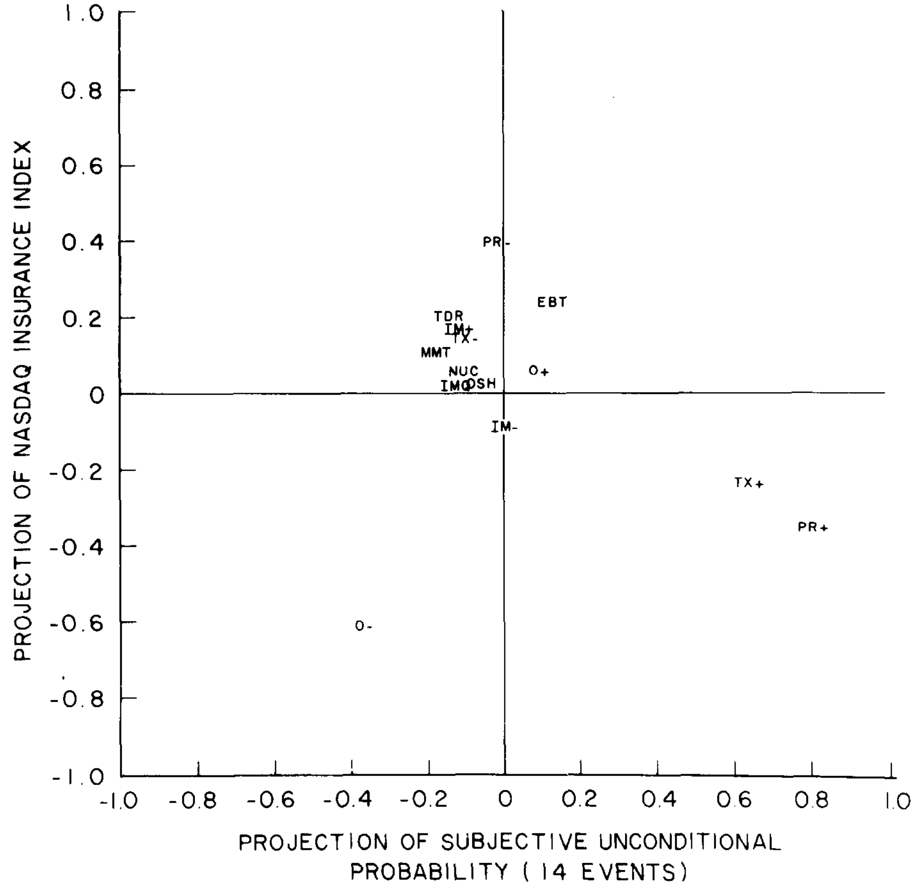

Fig. 8. Impact plot of projections of NASDAQ Insurance Index against projections for subjective unconditional probabilities, using INDSCAL solution.

The following procedural points should be noted. First, if our panelists had been actively engaged in trying to influence the probabilities of one or more of the events, then vectors for scenarios alternative to the one judged most likely could have replaced the SUCP data vectors graphically portrayed in Figures 6, 7, and 8. A second point is that the investigators designing an impact scaling study can also devise their own composite index, using a weighted average of the (separate) business parameters to get a summary description of the outlook for the market (or whatever interest, corporate or otherwise, is of concern). Assignment of weights in such a composite index necessitates decisions about tradeoffs among potentially conflicting or incompatible goals and consideration of willingness to accept the ensuing risks. Such a (weighted) composite average constitutes, in effect, a utility function aggregated over the separate indices of stock market behavior. We have not included such a composite in our study because a) as noted at the bottom of Table 2, we already have several widely known composite indices and b) Figures 6 and 7 already give a good indication of the direction in which the corresponding vector would lie, unless a pathological weighting scheme were adopted. In the absence of these two considerations, practitioners of any practical application would be well advised to include one or more such composites, particularly as ways of gauging the possible consequences of risky decisions involving tradeoffs among corporate goals.

TABLE 3
INDCLUS Solution for the 14 Economic Events[a]

Cluster	Weight	Probability	Events Contained in Cluster					
1	0.425	0.871*	PR−	TX−	IM+	O−	O+	
			NUC	TDR	MMT	OSH	IMQ	EBT
2	0.133	0.092*	PR−	IM+	MMT	EBT		
3	0.132	0.071*	PR+	TX+	IM−	O+	NUC	MMT
			OSH	IMQ	EBT			
4	0.129	0.457*	TX−	O−	NUC	MMT	OSH	IMQ
5	0.124	0.107	TX−	NUC	TDR	MMT	IMQ	EBT
6	0.109	0.005*	PR+	PR−	TX+	TX−	O+	TDR
			OSH	IMQ	EBT			
7	0.101	0.089	TX+	IM−	OSH	IMQ		

[a]These seven clusters, plus an additive constant of 0.085, accounted for 33.4% of the variance. See text for interpretation of clusters and Table 1 for explanation of codes used for events. Probabilities were estimated as cosines using PROFIT, and the values with asterisks apply to the complements of respective clusters.

The same 58 matrices used as input to generate Figures 2–5 were also used to obtain the INDCLUS solution in Table 3.The seven-cluster solution is presented along the weights for the aggregate data set; otherwise, it would be necessary to list the 7 × 58 = 406 weights fitted for each cluster in each matrix. This solution shows considerable overlap among the seven clusters and accounts for 33.4% of the variance present in the 58 matrices, roughly the same VAF as the four-dimensional INDSCAL solution presented earlier.

Considering the clusters by descending order of weights, we note that the first cluster comprises all the events except increases in income tax and the prime lending rate. Viewed from a conservative political stance, these two events are often dreaded and viewed as choking the economy. The events that are included in the cluster span issues of regulation, technology, energy, and transportation. The four events in the second cluster (drop in prime rate, more funding for mass transit, electronics breakthrough, and more money for IMF) would stimulate industrial growth in the United States and elsewhere. Interpretation of the third cluster is very problematic, and we do not have a parsimonious explanation.

The fourth cluster can be viewed as governmental responses (e.g., relaxation of OSHA rules, more money for mass transit) to such crises as an oil shortage. A tax break and the technological advances in cluster five are the types of events usually associated with a booming economy. Note that the governmental actions included here would be expected to stimulate U.S. industrial growth. The sixth cluster emphasizes government options in regulation, taxation, and monetary issues. The inclusion of an oil surplus, however, is surprising. The final cluster can be viewed as the scenario for the government raising money through increased taxation in an express effort to strengthen U.S. industry.

We noted earlier that this INDCLUS representation in Table 3 could be viewed as a seven-dimensional spatial solution, with each event having a coordinate of 1 (if included in the cluster now viewed as a spatial dimension) or 0. Because such a representation is not easily visualized, embedding the clusters in the first two dimensions of the SINDSCAL solution for the 14 events, as in Figure 9, may be more easily interpreted. Note that the axes have been permuted from their order in Figure 2 so that the clusters fit more easily in the available space. Also, there is no contour to represent the sixth cluster simply because its inclusion would have made Figure 9 too cluttered.

Although a seven-dimensional spatial solution with binary coordinates is not easily visualized, it is appropriate for embedding the vector of subjective unconditional probabilities ("scenario judged most likely") to predict the probabilities of the various clusters as discrete "superevents." The program PROFIT (Chang and Carroll [10]) provides the

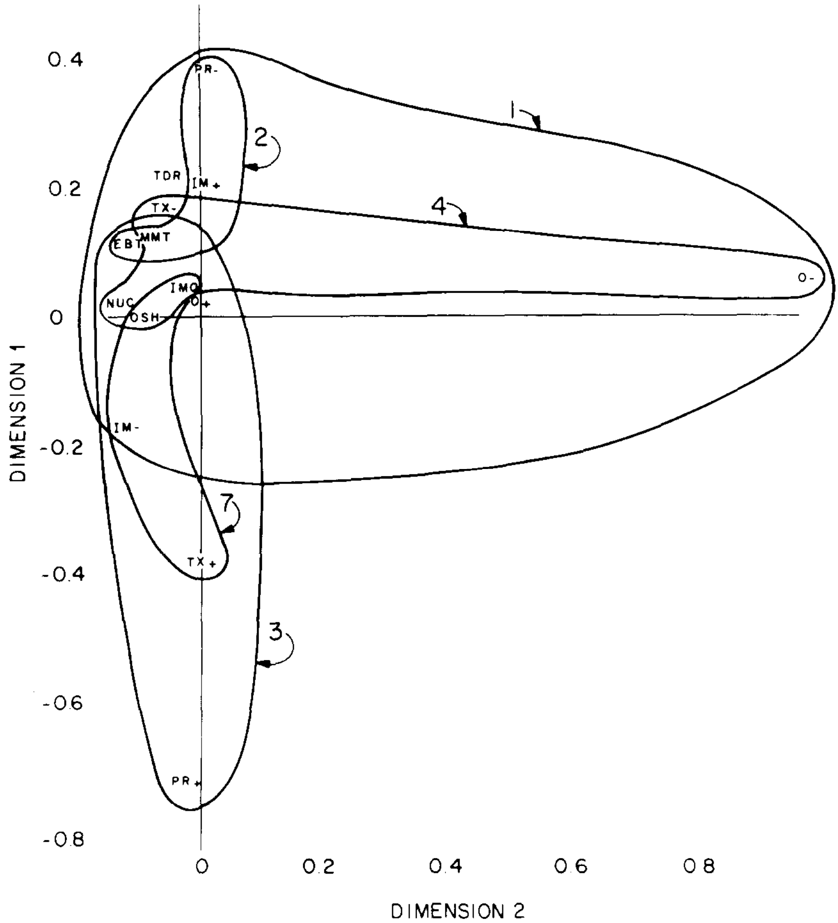

Fig. 9. INCLUS event clusters embedded in first two dimensions of INDSCAL event space. (Note that axes are reversed from Figure 2.)

direction cosines for fitting the scenario judged most likely (as well as the vectors for each of the fifteen stock market indices). In Table 3, these cosines are given as probabilities for the clusters or, where asterisks are included, for the complement of the respective cluster. (The complement arises when the estimated cosines are negative.) For example, the model predicts that, according to the panelists' judgments, the probability of increases in income tax and the prime rate (viz., the two events not included in the first cluster) is 0.871. Similarly, the probability of the scenario suggested above for the events in cluster seven is 0.089. (It should be noted that these probability estimates assume that with respect to causality, the clusters are mutually exclusive.) If the INDSCAL dimensions presented in Figures 2 and 3 are regarded as superevents, then the probabilities corresponding to dimensions one through four are 0.765, 0.258, 0.221, and 0.546. But these latter probabilities apply to *continuous* dimensions, in contrast to the probabilities in Table 3, where an event either is or is not (with no continuous intermediate gradations) relevant to a given superevent, to which the corresponding probability is assigned.

To offer a final comparison of INDSCAL and INDCLUS as vehicles for impact scaling, we note that the impact plot for the NASDAQ Insurance Index, using the INDSCAL solution and presented in Figure 8, has its INDCLUS counterpart in Figure

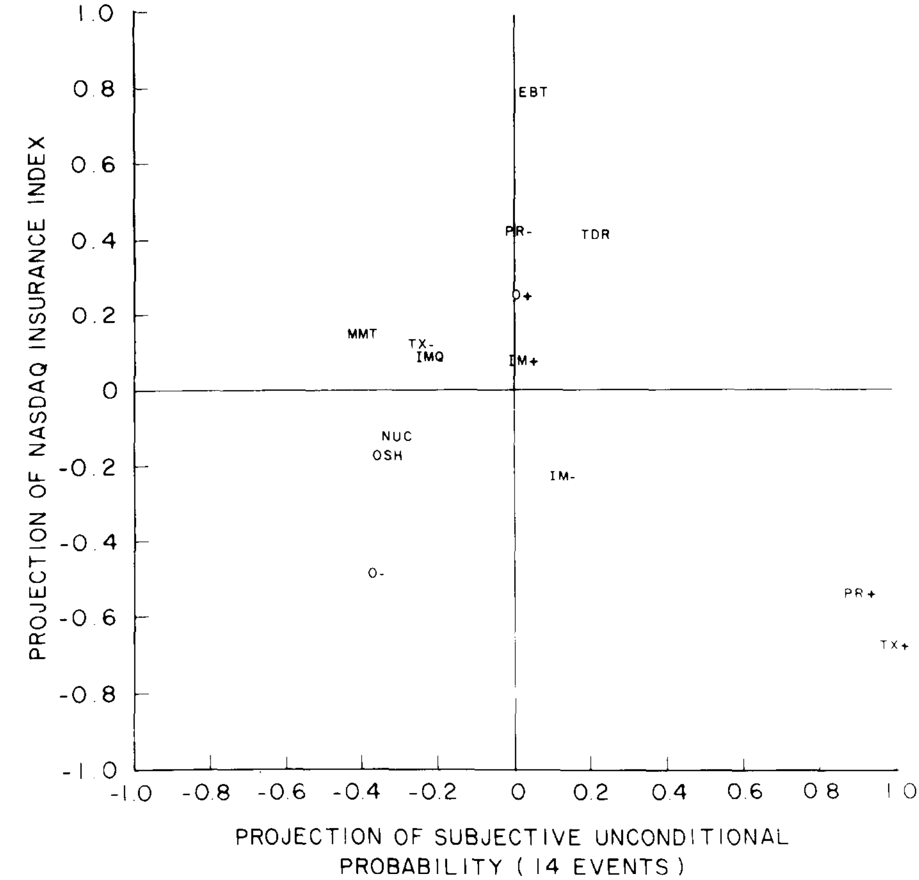

Fig. 10. Impact plot of projections of NASDAQ Insurance Index against projections for subjective unconditional probabilities, using INDCLUS solution.

10. That is, the projections being plotted in Figure 10 are taken from the INDCLUS solution, regarded here as a seven-dimensional space with binary coordinates. As in Figure 8, increases in the prime rate and income tax are events whose probabilities should be decreased in order to maximize linear correlation and increase the NASDAQ Insurance Index. An interesting detail of Figure 10 not found in Figure 8 is that more funding for mass transit is an event whose probability should be increased to maximize the NASDAQ Insurance Index. Given the highly publicized misgivings of many insurance companies to provide automobile policies and coverage, this result seems noteworthy.

We noted in the discussion of Figure 8 that planners having actual resources to influence the outcomes of various events could revise the probabilities constituting the "scenario judged most likely" and represented by the vector labeled SUCP (subjective unconditional probabilities) in Figures 6 and 7. As an example of this procedure, we return to the example of the NASDAQ Insurance Index. Consider as a *hypothetical* example a situation where the Federal Reserve Board might wish to bolster a sagging value for this index. Clearly, that Board can influence the probabilities of changes in the prime lending rate and conceivably might be able to influence the probability of governmental agencies providing more funding for mass transit. Accordingly, we have changed the following three unconditional probabilities listed in Table 1 to obtain a new "opti-

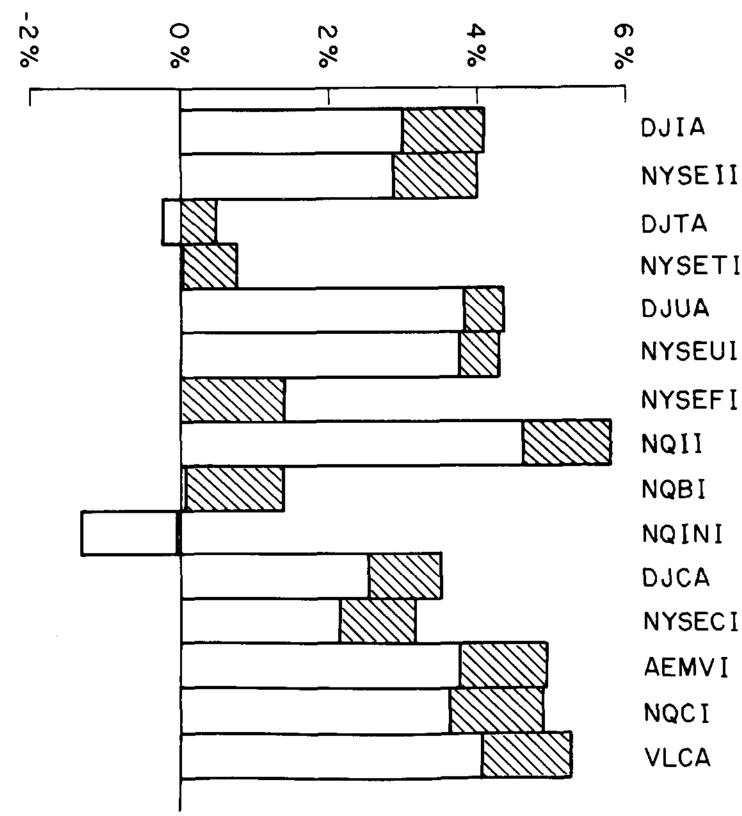

Fig. 11. Histograms for expected changes (in percentage points) in the indices as a function of the scenario judged most likely and of the optimistic scenario. The latter are represented by bars filled with diagonal lines.

mistic" scenario for the NASDAQ Insurance Index: the probability of an increase in the prime interest rate goes from 0.738 to 0.25; a decrease in the prime rate goes from 0.302 to 0.60; and more funding for mass transit goes from 0.209 to 0.50. These three altered values plus the original values for the remaining 11 events constitute the probabilities whose corresponding vector is labeled SUCP1 in Figures 6 and 7 and which is interpretable as an optimistic scenario.

In either Figure 8 or 10, the effects of redefining probabilities of these events can be seen by simply adjusting the abscissae of the points representing these three events. To go a step further, eq. (27) can be used to estimate the predicted change for each of the 15 indices, using either the SUCP or the SUCP1 values. (Either the INDSCAL or the INDCLUS solution could be used for this purpose; our illustration below uses the latter.) Following Sen (personal communication) and Bozzomo [2], we are using a histogram to present the expected changes in the fifteen parameters, as a function of the two different scenarios. Note that, in Figure 11, the two indices deemed to drop the most in the scenario judged most likely (SUCP) were the Dow Jones Transportation Index (−2.5%) and the NASDAQ Insurance Index (−1.3%). Under the optimistic scenario, the former index would now *increase* by 4.8% and the latter would drop by only 0.04%. All the other indices would increase as well. The complete absence of any tradeoff among indices suggests that additional indices (e.g., to gauge inflation, or other such economic factors more remote from these rather narrowly defined stock market indices) not included in the present study would also have to be considered. (Otherwise, one assumes that the obviousness and simplicity of this strategy would have already caused it to be adopted.) Again, we would like to emphasize that the preceding illustration was hypothetical and was intended only to give a concrete example of how impact scaling can be used for managerial planning.

In summary, we advocate the use of both the INDSCAL and INDCLUS models for representing the data required for impact scaling, since the models offer complementary ways of representing structure of the events. Another point we have not emphasized is that impact scaling can be used iteratively in attempting to reach greater convergence and consensus of the panelists' views, or, alternatively, to explore the nature of and reasons for important systematic discrepancies and divergencies in these points of view. Limitations of time and expense prevented us from doing so, but with an intact group of panelists all employed within the same organization, these difficulties become less prohibitive.

Future Prospects

For improving the quality of group judgment (Einhorn, Hogarth, and Klempner [18]), a number of strategies have been advanced for weighting subjective probability estimates from different panelists (e.g., Chernous'ko [11], Freeling [22], Morris [31], and Shneiderman [43]). Similarly, Keeney [28, p. 828] called for greater attention to individual differences among panelists.

We noted earlier that both the INDSCAL and INDCLUS models are designed to depict such differences among panelists. In the discussion of Figure 4, we observed revealed differences among groups of the panelists in that INDSCAL solution. Although somewhat more difficult to present in this paper, a similar presentation could have been offered for the INDCLUS solution. In addition to the facility these models offer for depicting individual differences among panelists, it should be apparent that weights for panelists could easily be introduced in eqs. (15) and (16), defining the impact matrices. (The INDSCAL analysis equated the variances of each of the 58 impact matrices, but the INDCLUS analysis used no such standardization.) Such weights could implement any of various schemes for weighting panelists' probability estimates, and represent a next step for developments in Impact Scaling.

Yet another application of the INDCLUS model to the psychology of judgment and subjective probabilities concerns one of the better known and most elaborately formalized approaches to "vague probabilities" (i.e., probabilities as interval rather than point estimates) given by Shafer's [39, 40] approach to belief functions. A terse summary begins with the observation that empirical evidence rarely bears a one-to-one relation with a single event, but is instead associated with a set of X events. Then a panelist's "belief function" for a given item of evidence becomes a function of the probability distribution over the power set of X. As Freeling [23, pp. 4.3–4.9] notes: "After the receipt of a given piece of evidence we model the entire belief structure, by assessing the basic probability assignment over every subset of X . . . [T]he basis of the theory is the probability assignment m, which is simply a distribution over [all] the solutions of X. In fact, m defines a random subset of X." Given this framework, the next step is to assume a convenient distribution of probabilities over all subsets of X and proceed from there.

Shafer's [39, 40] approach is highly ingenious and possesses great formal elegance. However, formidable difficulties arise in trying to apply Shafer's theory to real situations (cf., Freeling [23, p. 4.12]). We would like to suggest that INDCLUS offers a way to render Shafer's approach applicable to date from empirical problems. Concretely, we being by noting that Shafer's dependence on the set X ("evidence in support of ________") could be obtained from panelists. *Then instead of looking at the power set of X, we need look only at the INDCLUS-supplied clusters involving X.* That is, instead of looking at random subsets of X, we would look at rationally obtained subsets (clusters), based on panelists' data. Shafer's preoccupation with the power set of X is reasonable enough from the standpoint of formal elegance, but immediately isolates the theory from data-oriented tests. We propose to make the problem more tractable by considering only those subsets of X that are relevant to the subjects' perceptions of evidence. INDCLUS, through the use of impact scaling, offers the vehicle for winnowing the power set of X to a manageable size.

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