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Regular blockmodels of multiway, multimode matrices *

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Blockmodels are used to collapse redundant elements in a system in order to clarify the patterns of relationships among the elements. Traditional blockmodels define redundancy in terms of *structural equivalence*. This choice serves many analytic purposes very well, but is inadequate for others. In particular, role systems would be better modeled by blockmodels based on *regular equivalence*. The first goal of this paper is to generalize blockmodels to incorporate both structural and regular equivalence. Another limitation of traditional blockmodels is that they are defined only for (collections of) 2-way 1-mode adjacency matrices. This excludes common datasets such as actor-by-event, actor-by-organization, item-by-use and case-by-variable matrices. It also excludes 3-way data such as actor-by-actor-by-time or subject-by-verb-by-object matrices. The second goal of this paper is to define blockmodels for multiway, multimode matrices in general. In so doing, we also shift the focus of attention away from the blocking of actors (or other entities) and toward the blocking of ties (or multiway cells).

An important tool in the analysis of social networks is the *blockmodel* (Breiger *et al.* 1975; White and Breiger 1975; White *et al.* 1976; Boorman and White 1976). Blockmodeling seeks to (a) cluster actors who have substantially similar patterns of relationships with others, and (b) interpret the pattern of relationships among the clusters. Viewed as a method of data reduction, blockmodeling is a valuable technique in which redundant elements in an observed system are reduced to yield a simplified model of relationships among types of elements.

However, despite initial papers presenting the method in this light (Breiger *et al.* 1975; Boorman and White 1976), blockmodeling is not

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* This paper is based on Chapter 3 of Borgatti (1989).

usually seen this way. Rather, blockmodeling is seen as a tool for discovering roles and positions occupied by actors in a social structure (Knoke and Kuklinski 1982). This is unfortunate because standard blockmodeling is not well suited to this task.[1] Standard blockmodels place actors in the same block if they are relatively *structurally equivalent* (Lorrain and White 1971; Burt 1976). That is, actors are blocked together if they have the same types of ties with the same others. These blocks are then interpreted as positions, and the modal relationships observed between members of a given position and members of other positions are interpreted as the role played by actors occupying that position.

However, as many authors have pointed out (Sailer 1978; White and Reitz 1983; Winship and Mandel 1983; Doreian 1988a, 1988b; Faust 1988; Pattison 1988; Borgatti and Everett 1989, 1992), these interpretations do not fully capture the notions of position and role. In reality, actors occupying the same position (such as two doctors or two mothers) do not necessarily have similar ties with the *same* others. Rather, they have the same ties with the same *types* of others. That is, all doctors may have the same role-relations with patients, nurses, suppliers and other doctors, but not necessarily the same patients, nurses etc. Similarly, two mothers do not normally have the same children: it is enough that they have any children.

Thus, structural equivalence is not an appropriate basis for classifying actors into social positions. A better choice is *regular equivalence* (White and Reitz 1983), as many authors have argued (Doreian 1988a, 1988b; Faust 1988; Borgatti and Everett 1989). Under regular equivalence, equivalent actors have the same types of ties not with the same actors but with equivalent actors. This conception more accurately captures the concept of role and position. The first goal of this paper

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[1] Except when used simply to reduce the data prior to a different kind of role analysis. This is the approach taken by Boorman and White (1976) and Wu (1983). After collapsing redundant actors, they analyze compound relationships among relations. For example, Boorman and White study positive and negative relations in the Sampson (1968) monastery data. They find (pp. 1393–1394) that the compound relation "friend of an enemy" mirrors the simple relation "enemy of". These findings are revealing of the social roles of "friend" or "enemy", and so this approach is indeed a role analysis. However, blockmodeling has little to do with it: it is used merely to collapse substantially redundant actors in order to simplify the analysis and possibly eliminate some effects of measurement and other error. In later work this step is dispensed with (Breiger and Pattison 1986), and compound relations are computed from the raw data.

is to generalize the notion of blockmodels to utilize regular equivalence as the basis for blocking.

The second goal of this paper is to enhance the usefulness of blockmodeling as a data reduction tool for many different kinds of datasets. Until now, blockmodeling has been used only in the context of network analysis, although the algorithms behind it have long been used in other fields (for a review, see Arabie *et al.* 1978). In this paper, we extend blockmodeling to apply not only to network data, but to any data that can be represented as a matrix. This makes it possible to blockmodel actor-by-event, actor-by-organization, item-by-use, consumer-by-product and other common incidence matrices. Further, following Baker (1986) we extend blockmodeling beyond the confines of 2-way matrices to multiway, multimode data. Applications include actor-by-actor-by-relation data, actor-by-means-by-outcome matrices, and so on. In addition, we define blockmodels for both binary and valued data, including nominal-scale (categorical) values.

1. Multiway, multimode matrices

Network data typically consist of one or more binary *adjacency* matrices representing social relations among a set of actors. Adjacency matrices have two *ways*, corresponding to their rows and columns. A way is a set of objects that subscript or index the cells of a matrix.

An adjacency matrix is usually thought of as having one *mode*, corresponding to the single set of actors that both the rows and columns reference. A mode is a set of objects indexed by a matrix way (Tucker 1964). In contrast, an *incidence* matrix that records which women (rows) attended which events (columns) has two ways (rows and columns) and two modes (women and events). A matrix of distances between pairs of cities has one mode because both ways refer to the same objects – in this case cities. A 2-way matrix must be square to have only one mode.

A mode may be viewed as a *kind* of way. The set of modes of a matrix partitions or maps the set of ways of a matrix into a set of types or classes of equivalent ways. For example, in the case of a 2-mode, 3-way matrix such as a collection of actor-by-actor adjacency matrices, each representing a different social relation, we implicitly map the first two ways onto the first mode (actors) and the third way onto the second mode (relations). We regard two ways as *distinct* if they are mapped to different modes. Two ways must be distinct if they contain different numbers of elements.

Whereas the number of ways in a matrix is unambiguous and clearly a property of the matrix, the same cannot be said for modes. Data analysts may reasonably disagree on the number of modes in a given matrix. Furthermore, the same analyst may choose to define the number and make-up of modes of a given matrix differently at different times depending on the analytic question to be answered. For example, for some purposes, the non-symmetric adjacency matrix of a directed graph may be regarded as having one mode or two, depending on the analytic purpose. Thus modes are most properly thought of as properties of the analysis. That said, however, it must be admitted that, for most datasets, there is a clear and natural choice of modes. For that reason, we shall find it convenient to refer to "2-mode matrices" rather than a "2-mode analysis of a matrix".

A 2-way matrix consists of a set of rows and a set of columns, a set of cells (i, j), and a set of values or entries in those cells. More abstractly, the matrix consists of a pair of ways $[R, C\}$, the cartesian product $R \times C$, and an assignment of a value to each ordered pair $(r, c) \in R \times C$. Hence, a matrix may be viewed as a mapping of the cartesian product of a set of ways into the set of real numbers. We use this conception to define the general notion of a multiway matrix:

Definition 1. Let $W = \{W_1, W_2, \ldots, W_m\}$ be a set of m ways. Let $E \subseteq \Pi W_i$ for $i = 1, \ldots, m$ be a subset of the cartesian product of all ways. An *m-way matrix* $X(W, E)$ is a function $X\colon E \to \boldsymbol{R}$. The *cells* of an m-way matrix are the m-tuples $c = (c_1, \ldots, c_m)$ where $c_i \in W_i$. The *value* of a cell c is written $x(c)$ or $x(c_1, \ldots, c_m)$, as convenient.

Definition 1 allows for the possibility that some cells of a matrix (those not in E) have *missing*, *null* or undefined values. This permits a distinction between the quantity zero and the absence of a value. This is particularly useful when using matrices in which cell values are measured on an interval scale in which a value of zero implies not the absence of something but a quantity one unit greater than -1. It is also useful when using matrices to represent valued graphs in which the value assigned to a pair of unconnected nodes is undefined rather than zero. For example, in a graph in which links represent pipes and values represent the temperature sensitivity of each pipe, it makes no sense to measure the temperature sensitivity of non-existent pipes, and so no values are observed for pairs of unconnected nodes. Similarly, in a matrix representing a non-reflexive relation such as "is married to", it is desirable to distinguish cells in which the measured relationship could conceivably occur (all off-diagonal cells) from cells in which the relationship could not possibly occur (diagonal cells). The distinction corresponds roughly to the distinction made in other contexts (cf. Bishop *et al.* 1975) between "structural zeros" and "data zeros".

2. General blockmodels

As presented by Breiger *et al.* (1975) and White *et al.* (1976), a blockmodeling analysis begins, in the simplest case, with a matrix X in which rows and columns refer to actors. The matrix represents an observed social relation such as "is a friend of" or "reports to", and has values $x(i, j) = 1$ if actor i has the specified relationship with actor j, and $x(i, j)$ is null otherwise. An example is given in Figure 1.

Blockmodeling can be divided into two distinct steps, which we shall refer to as *blocking* and *modeling*, respectively. Traditionally, the blocking step consists of partitioning actors in a network into structurally equivalent sets called *blocks*. For example, a blocking of the matrix in Figure 1 yields the following blocks of actors: {a, c}, {b, d},

		Actor									
		a	b	c	d	e	f	g	h	i	j
	a		1	1	1						
	b	1		1		1					
	c	1	1		1						
	d	1		1		1					
Actor	e		1		1		1				
	f					1		1		1	
	g						1		1		1
	h							1		1	1
	i						1		1		1
	j							1	1	1	

Fig. 1. An adjacency matrix.

		Actor									
		A		B		C	D	E		F	
		a	c	b	d	e	f	g	i	h	j
Actor A	a		1	1	1						
	c	1		1	1						
B	b	1	1			1					
	d	1	1			1					
C	e			1	1		1				
D	f					1		1	1		
E	g						1			1	1
	i						1			1	1
F	h							1	1		1
	j							1	1	1	

Fig. 2. Blocked matrix based on a structural blockmodel of the matrix in Figure 1.

$\{e\}$, $\{f\}$, $\{g, i\}$, $\{h, j\}$. For future reference, we define a blocking formally as follows:

Definition 2. Let $X(W, E)$ be an m-way matrix. A *blocking* of X is a set of partitions $P = (P_1, \ldots, P_m)$ of each of the ways of X.

A blocking is said to be trivial if each element of each way is placed in a separate block (the *identity* blocking), or if all elements of each way are placed in the same block (the *complete* blocking). Two elements u, $v \in W_i$ of a given way are members of the same *block* (i.e., are equivalent) if $P_i(u) = P_i(v)$. Figure 2 presents the *blocked matrix*, which is a rearrangement of the rows and columns (and other ways) of the data matrix according to blocks. Note that the partitions of the ways of a matrix induce a partition of the cells as well: cells are equivalent if and only if they connect equivalent way-elements. The classes of equivalent cells, together with their values, are referred to as *matrix block*. In Figure 2, thirty-six matrix blocks are evident. For future reference, we define matrix blocks as follows:

Definition 3. Let $X(W, E)$ be an m-way matrix. Let P be a blocking of X. Then P induces an equivalence relation P^* on ΠW where, $\forall c$, $d \in \Pi W \; cP^*d$ if and only if $P_i(c_i) = P_i(d_i)$ for $i = 1, \ldots, m$. An *m-way*

		Block					
		A	B	C	D	E	F
Block	A	1	1	0	0	0	0
	B	1	0	1	0	0	0
	C	0	1	1	1	0	0
	D	0	0	1	1	1	0
	E	0	0	0	1	1	1
	F	0	0	0	0	1	1

Fig. 3. Image matrix based on a structural blocking shown in Figure 2.

matrix block (or simply *matrix block*) is a submatrix of X whose cells comprise an equivalence class of P^*.

The modeling step consists of building a new matrix, based on the blocking, which summarizes the pattern of relationships observed in the data matrix. The new matrix, called the *image matrix* or *model*, has rows and columns corresponding to blocks and cell entries that indicate whether the actors belonging to the blocks are linked in the data matrix. An example is given in Figure 3. In the example, the (A, B) cell contains a value of 1 because, according to the blocking shown in Figure 2, all actors in the first block (A) are connected to all actors in the second block (B). In contrast, the (B, D) cell is empty because none of the actors in the second block (B) have any ties with any of the actors in the fourth block (D). A formal definition of the model will be given in a later section.

While some authors use the term "blockmodel" to refer to the image matrix alone, we prefer to think of the blockmodel as consisting of both the blocking and the image. Thus, we define a blockmodel as:

Definition 4. Let $X(W, E)$ be an m-way matrix. Let $P = \{P_1, \ldots, P_m\}$ be a blocking of X. Let I be an image matrix whose ways index the equivalence classes of the partitions in P. The collection $B(P, I)$ is a *blockmodel* of X.

We begin the discussion by considering alternative blockmodels constructed by different blocking criteria, then take up the question of appropriate modeling strategies.

3. Structural blockmodels

A blockmodel in the sense of Breiger *et al.* (1975) and White *et al.* (1976) is, in our terms, a blockmodel that uses structural equivalence as the basis for blocking. We refer to such a blockmodel as a *structural blockmodel*. Traditional blockmodels are defined for collections of 1-mode, 2-way matrices. Here we present a generalized definition for multiway, multimode matrices which includes the traditional definition as a special case.

Definition 5. Let $X(W, E)$ be an m-way matrix. A blockmodel $B(P, I)$ of X is a *structural blockmodel* if whenever $P_i(u) = P_i(v)$, u, $v \in W_i$ then $x(c) = x(d)$ for all cells c, $d \in E$ such that $c_i = u$, $d_i = v$, and $c_j = d_j$ for all $j \neq i$.

The definition states that two elements of a given way will be assigned to the same block if they have the same values across all matrix cells whose cell indices are the same for all other ways. For example, if elements 7 and 9 of the third way of a 5-way matrix are structurally equivalent, then $x(1, 2, 7, 6, 3) = x(1, 2, 9, 6, 3)$, and, more generally, $x(a, b, 7, d, e) = x(a, b, 9, d, e)$, for all elements of each of the other four ways.

The definition is made more comprehensible if we think of the elements of a given way as "choosing" the elements of other ways with certain intensities. For example, consider a crime-by-city-by-year matrix in which cell values record the number of crimes committed each year of a given type in various cities. We can think of a certain kind of crime as "choosing" a certain city and year with some frequency, and choosing another city and year with the same or different frequency. Similarly, we can think of a certain city as choosing to have a certain type of crime with a certain frequency in a given year. Put this way, the definition specifies that two crimes belong in the same block if they occur with the same frequencies in the same cities and years. At the same time, two cities belong to the same block if they have the same distribution of crimes in the same years, and two years are blocked together if they exhibit the same frequencies of crimes in the same cities.

Applied to a 2-way 1-mode matrix, Definition 5 reduces to the standard blockmodels proposed by Breiger *et al.* (1975) and White *et*

Corporate Board

		1	2	3	4	5
	1	1	1	0	0	0
	2	1	1	0	0	0
Person	3	1	1	1	1	1
	4	1	1	1	1	1
	5	0	0	1	1	1
	6	0	0	1	1	1
	7	0	0	1	1	1

Fig. 4. Structural blockmodel of a 2-way, 2-mode matrix.

al. (1976).[2] An example of a structural blockmodel is given in Figure 2. The image matrix is given in Figure 3. According to the blockmodel there are 6 distinguishable blocks of actors in the network: $\{a, c\}$ $\{b, d\}$ $\{e\}$ $\{f\}$ $\{g, i\}$ $\{h, j\}$. Since, aside from relationships with themselves, each pair of equivalent actors is connected and not connected to exactly the same others, there is in fact no reason to distinguish them. They are perfectly substitutable. Hence, the data reduction and consequent simplification provided by the image graph is gained with minimal loss of information.

Applied to a 2-way, 2-mode matrix, Definition 5 requires that two rows be placed in the same row-block if they have the same values across all columns. At the same time, two columns are placed in the same block if they have the same values across all rows. An example of a structural blockmodel of a 2-mode matrix is given in Figure 4. In the example, the rows correspond to businesspersons while the columns represent firms. The matrix records which persons sit on the board of directors of which firms. Two persons are perfectly structurally equivalent if they sit on exactly the same boards. Two boards of directors are perfectly structurally equivalent if they are composed of exactly the same persons.

An example of a 2-mode structural blockmodel of a *valued* matrix is given in Figure 5. In the figure, rows represent investment analysts while columns represent stocks, bonds or other securities. Equivalent

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[2] Our generalized definition does not repair a well-known shortcoming of the standard definition. When applied to directed graphs, this definition will find a pair of nodes structurally equivalent even if there is an unreciprocated arc from one "equivalent" node to the other. A correction based on a better definition of structural equivalence (Everett *et al.* 1990) should be used in actual computations. We omit the correction here in order to simplify the exposition.

		Security						
		1	2	3	4	5	6	7
	1	A	A	B	B	B	C	C
	2	A	A	B	B	B	C	C
Investment Analyst	3	A	A	C	C	C	C	C
	4	A	A	C	C	C	C	C
	5	A	A	B	B	B	B	B
	6	A	A	B	B	B	B	B

Fig. 5. Structural blocking of matrix recording risk ratings for seven stocks and bonds by six analysts. The values $\{A, B, C\}$ represent ordinal measurements with $A > B > C$.

analysts are those who give the same rating to the same securities. Equivalent securities are those that receive the same ratings from the same analysts. For datasets with just a few distinct values, as in this example; it is also possible to treat the different values as entirely different relations, creating a binary 3-way analyst-by-security-by-rating matrix. The advantages of blockmodeling the valued matrix directly are that it is simpler and it is readily extended to the case of continuous values.

Applied to a 3-way 2-mode matrix such as the actor-by-actor-by-relation data of Sampson (1968), Definition 5 reduces to Baker's (1986) extended blockmodel in which actors and relations are simultaneously partitioned. Actors are blocked together if they have ties to the same others on the same relations, and relations are blocked together if they connect exactly the same pairs of actors. Relations which are perfectly structurally equivalent relations are clearly redundant, and (all but one) can be ignored for all further analyses, including building role algebras (Boorman and White 1976; Sailer 1978). Thus, White and Reitz's (1983) theorem showing that the same semigroup is generated from a structural image matrix as from the raw data matrix is valid for the case when relations are blocked as well as actors.

4. Structural matrix blocks

The matrix blocks of a structural blockmodel have a characteristic appearance: all cells within a block have the same value (or are null). This applies both to binary and valued matrices, as the blockmodels in

Figures 4 and 5 demonstrate. Although obvious, this is a useful result which we state as a theorem for future reference.

Theorem 1. Let $B(P, I)$ be a blockmodel of a matrix $X(W, E)$. Then B is a structural blockmodel if and only if $x(c) = x(d)$ for every pair of cells c, d that are members of the same matrix block.

Proof. Suppose $B(P, I)$ is a structural blockmodel. Let $(c_1, \ldots, c_m)$ and $(d_1, \ldots, d_m)$ be two cells of the same matrix block Y. It follows that $P_1(c_1) = P_1(d_1)$ and since the blockmodel is structural $x(c_1, \ldots, c_m) = x(d_1, c_2, \ldots, c_m)$. It can easily be seen that $(d_1, c_2, \ldots c_m)$ is also a member of Y and since $P_2(c_2) = P_2(d_2)$ we can similarly deduce that $x(d_1, c_2, \ldots, c_m) = x(d_1, d_2, c_3, \ldots, c_m)$. Continuing in this manner we eventually produce the sequence $x(c_1, \ldots, c_m) = x(d_1, c_2, \ldots, c_m) = x(d_1, d_2, c_3, \ldots, c_{\mathrm{m}}) = \ldots = x(d_1, \ldots, d_m)$. Conversely, suppose that for every pair of cells c, d in a matrix block $x(c) = x(d)$. Then it follows directly from the definitions that $B(P, I)$ is structural.

The importance of this theorem is that it provides a basis for a measure of blockmodel fit. A blockmodel is structural to the extent that its matrix blocks have uniform values. A natural measure of fit, then, is the average variance within matrix blocks. In the case of the adjacency matrix of a non-valued graph, this criterion is closely related to the objective function proposed by Batagelj *et al.* (1992ue) for minimization by a structural blocking algorithm. The advantage of the average variance criterion is that it is appropriate for both binary and valued data. A block modeling algorithm using this criterion has been implemented in the UCINET IV (Borgatti *et al.* 1991) network analysis software.

5. Regular blockmodels

A regular blockmodel differs from a structural blockmodel in the criteria used to assign actors to blocks. Whereas structural blockmodels are based on structural equivalence, regular blockmodels are based on the more general notion of regular equivalence. A regular blockmodel is defined as follows:

Actor

		a	c	h	j	b	d	g	i	e	f
	a		1			1	1				
	c	1				1	1				
	h				1			1	1		
	j			1				1	1		
	b	1	1							1	
Actor	d	1	1							1	
	g			1	1						1
	i			1	1						1
	e					1	1				1
	f							1	1	1	

Fig. 6 Regular blocking of matrix shown in Figure 1.

Definition 6. Let $X(W, E)$ be an m-way matrix. A blockmodel $B(P, I)$ of X is a *regular blockmodel* if whenever $P_i(u) = P_i(v)$ u, $v \in W_i$ then for all cells $c \in E$ and $c_i = u$ there exists a cell $d \in E$ such that $d_i = v$, $x(c) = x(d)$ and cPd.

Applied to ordinary network data (2-way, 1-mode adjacency matrix) in which ties are coded "1" if present and null if absent, Definition 6 reproduces exactly the regular equivalence definition of White and Reitz (1983). If a blocking is regular, then if any actor in a given block A has a tie with any actor in a given block B, then all the actors of A have a tie with some actor of B, though not necessarily the same actor. An example of a regular blockmodel of the 2-way 1-mode matrix in Figure 1 is given in Figure 6. The image matrix is given in Figure 7. According to the blockmodel, there are three distinguishable types of actors in the network: $\{a, c, h, j\}$ $\{b, d, g, i\}$ $\{e, f\}$.

Applied to a 2-way, 2-mode matrix, Definition 6 requires that two rows placed in the same row-block have the same values across all columns. At the same time, two columns are placed in the same block

	ac hj	bd gi	ef
achj	1	1	
bdgi	1		1
ef		1	1

Fig. 7. Image matrix for regular blocking shown in Figure 6.

		Restaurant				
		1	2	3	4	5
Customer	1	1	1	1		1
	2	1	1		1	
	3	1			1	1
	4	1		1		1
	5		1		1	
	6	1	1			
	7	1	1			

Fig. 8. Two-way 2-mode data matrix in which $x(i, j) = 1$ if person i visits restaurant j during a given time period.

if they have the same values across all rows. As an example, consider the customer-by-restaurant matrix X shown in Figure 8, which records which persons visit which restaurants during a given time period. Thus $x(i, j) = 1$ if person i visits restaurant j, and $x(i, j) = \text{null}$ otherwise. According to Definition 6, two persons are equivalent if they visit equivalent restaurants. A regular blocking of the matrix is presented in Figure 9. An image matrix is presented in Figure 10. Clearly, the regular blocking has identified two types of customers (labeled X and Y) as well as two types of restaurants (labeled A and B). The customers in X differ from the customers in Y in that X customers visit both kinds of restaurants, but Y customers only visit B restaurants. Correspondingly, A restaurants differ from B restaurants in that they are visited only by X customers whereas B restaurants are visited by both kinds of customers. In a sense, the X role entails the Y role, and, symmetrically, the B role entails the A role.

Let us assume that the reason the data turn out the way they do is that the A restaurants are expensive while the B restaurants are

			Restaurant				
			A		B		
			1	2	3	4	5
Person	X	1	1	1	1		1
		2	1	1		1	
		3	1			1	1
		4	1		1		1
		5		1		1	
	Y	6			1		1
		7				1	

Fig. 9. Regular blockmodel of matrix in Figure 8.

	A	B
X	1	1
Y		1

Fig. 10. Image matrix for regular blocking shown in Figure 9.

cheap, and the X customers are wealthy while the Y customers are poor. Wealthy customers can visit either kind of restaurant, but poor customers are restricted to cheap restaurants. It is interesting to note that computing a structural blockmodel of this or any other data generated according to the same underlying rules would fail to reveal the existence of these basic subdivisions. The reason is that there is nothing in the underlying situation that would cause all poor people to visit exactly the same restaurants and all rich people to visit (a different set of) exactly the same restaurants, as would be necessary for a structural blockmodel to identify the groups. After all, the data may have been collected from people living in completely different parts of the globe. Further, there is no reason why each rich person must visit *every* single restaurant and each poor person must visit every single cheap restaurant. Yet that is what would be required in order for a structural blockmodel to detect the underlying organizing pattern.

Definition 6 applies to valued matrices as well. As an example, consider the customer-by-restaurant matrix X given in Figure 11. Matrix values $x(i, j)$ give the occasion (lunch or dinner) for which person i is most likely to eat at restaurant j. A 2-mode regular blockmodel will consist of two partitions, one of the customers (rows)

		Restaurant				
		1	2	3	4	5
Person	1	1	1	1	2	1
	2	1	1	2	1	2
	3	1	2	2	1	1
	4	1	2	1	2	1
	5	2	1	2	1	2
	6	1	1	2	2	2
	7	1	1	2	2	2

Fig. 11. Valued 2-way 2-mode data matrix in which $x(i, j) = 1$ if person i visits restaurant j for lunch and $x(i, j) = 2$ if i visits j for dinner.

			Restaurant				
			A		B		
			1	2	3	4	5
Person	X	1	1	1	1	2	1
		2	1	1	2	1	2
	Y	3	1	2	2	1	1
		4	1	2	1	2	1
		5	2	1	2	1	2
	Z	6	1	1	2	2	2
		7	1	1	2	2	2

1 = Lunch
2 = Dinner

Fig. 12. Regular blocking of the matrix in Figure 11.

and one of the restaurants (columns). First consider the customer partition. Under what conditions would customers *u* and *v* be classed together? The definition says that for every restaurant that customer *u* visits, *v* must visit an equivalent restaurant on the same occasion. Similarly, for two restaurants *p* and *q* to be classed together, it must be true that for every customer that visits restaurant *p*, there is an equivalent customer who visits restaurant *q* on the same occasions.

The blockmodel in Figure 12 is regular. In this particular blockmodel, it happens that restaurants fall into two groups (labeled A and B) and customers fall into three groups (labeled X, Y and Z). The A-class restaurants differ from the B-class in that two different types of people (X and Z) visit the class A restaurants only for lunch, whereas no segment does that with the class *B* restaurants. On the contrary, at least one group (Z) visits the Bs only for dinner. Turning to the customers, it is clear that two of the groups (X and Z) distinguish between restaurants on the basis of occasion, but one group, (Y) does not.

Note that the number of restaurants visited by any person for a given kind of meal is not important. For example, persons 4 and 5 are equivalent, yet person 5 visits two class *B* restaurants for dinner while person 4 visits only one.

It should also be noted that the data shown in Figure 8 and Figure 11 do not differ except that nulls in the former matrix are recoded as twos in the latter matrix. Yet the resulting blockmodels (Figures 9 and 12) are different. In the first blockmodel, customers 1 through 5 are all seen as one block, whereas in the second, customers 1 and 2 are distinguished from 3, 4 and 5. If we interpret ones in Figure 11 as meaning "visits" and twos as meaning "does not visit", then the difference in blockmodels is, at first glance, puzzling. The reason is that using a one-two coding (versus a one-null coding) puts visiting and not-visiting on an equal footing; both are legitimate relationships that a customer might have with a restaurant and which might distinguish that customer from another customer. In Figure 12, customers 3, 4 and 5 resemble each other (and differ from 1 and 2) in that they visit and fail to visit the same types of restaurants. Customers 1 and 2 visit all A restaurants. They do not have a "not visit" relationship with any A restaurants. In contrast, customers 3, 4 and 5 have a "not visit" relationship with both A and B restaurants.

Thus, regular blockmodels give the analyst the option of choosing to regard the absence of a tie as a positive statement about the relationship between two entities. This means that regular blockmodels can be used to correct a deficiency noted by Borgatti and Everett (1989) in the concept of regular equivalence, which is that a partition that is regular on a graph is not necessarily regular on its complement. In other words, recoding the values of an adjacency matrix so that nulls become ones and ones become nulls, can yield rather different results. By choosing to code nulls as zeros (or any other valid value), regular blockmodels can be obtained which hold simultaneously for a graph and its complement.

It should be noted that the fact that regular blockmodels are defined for valued as well as binary matrix is a convenience but not a necessity. This is because any valued m-way matrix can always be represented as an $(m+1)$-way matrix with value as the additional way. From a computational point of view, of course, such a representation is significantly less desirable than the more compact valued representation (particularly if there are many distinct values). However, from a conceptual point of view, the binary representation has the benefit of yielding blockmodels which directly partition not only the ways of the valued matrix, but also the values themselves.

6. Regular matrix blocks

Regular blockmodels have a characteristic appearance. Consider the blockmodel in Figure 12. The row and column partitions induce a partition of matrix cells into six matrix blocks, which we shall refer to as XA, XB, YA, YB, ZA, and ZB. Each of the six matrix blocks is either filled with all the same value (as are XA, ZA and ZB), or each row and column of the matrix contains at least one 1 and one 2. The general principle is that if any cell in a matrix block contains a given value, then every row and column in that block must contain at least one instance of that value [3]. Any matrix block that has this property is called a *regular matrix* block. In a regular blockmodel, every matrix block is regular. We state this result as a theorem for future reference:

Theorem 2. Let $B(P, I)$ be a blockmodel of an m-way matrix $X(W,E)$. If $c \in \Pi W$ then define $R_i(c) = \{d\colon\ d = (c_1, \ldots, c_{i-1}, e, c_{i+1}, \ldots, c_p),$ $P_i(e) = P_i(c_i),\ e \in W_i\}$. B is regular if and only if for every $c \in E$, gP^*c and $i = 1, \ldots, p$ there exists $h \in R_i(g)$ such that $x(c) = x(h)$.

Proof. Suppose B is regular. Let $c \in E$ and gP^*c. It follows that $P_i(g_i) = P_i(c_i)$ and therefore there exists an $h \in E$ such that $h_i = g_i$, $x(c) = x(h)$ and cP^*h. Since gP^*c and cP^*h, then gP^*h, so that $h \in R_i(g)$. Conversely suppose that $P_i(u) = P_i(v)$, $u,v \in W_i$ and let $c \in E$ with $c_i = u$ and $g = (c_1, c_2, \ldots, c_{i-1}, v, c_{i+1}, \ldots, c_m)$. Since gP^*c then there exists $d_1 \in R_1(g)$ such that $x(c) = x(d_1)$. The fact that $d_1 \in R_1(g)$ means that $d_1 = (e_1, c_2, c_3, \ldots, c_{i-1}, v, c_{i+1}, \ldots, c_m)$ with $P_1(e_1) = P_1(c_1)$ and so d_1P^*c; we can therefore apply the condition in the theorem to d_1 to obtain a $d_2 \in R_2(d_1)$ with $x(c) = x(d_2)$ and d_2P^*c. We can proceed in this manner and change every element of g except g_i to form a new cell d. From our construction $d_i = v$, $x(c) = x(d)$ and cP^*d and the theorem is complete.

According to the theorem, regular blockmodels can be defined in terms of the matrix blocks they induce: a blockmodel is regular if and only if every matrix block it induces is regular. The theorem may be viewed as expressing a principle of decomposability: if X is a matrix and there exists a blockmodel B of X that is regular, then X may be decomposed into a set of disjoint regular submatrices. Further, within any regular block we can search for non-trivial regular blockmodels. If any are found, the resulting partition can be regarded as a special kind of refinement of the original blockmodel, which we call a *local refinement*. If the same refinement is regular across all matrix blocks,

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[3] This observation first appears in an unpublished manuscript by D.R. White (1980: 29).

	1	2	3	4	5	6	7	8	9	10
1	1	1	2	2	2	2	1	1	2	2
2	1	1	2	2	2	2	1	1	2	2
3	2	2	1	1	2	2	2	2	1	1
4	2	2	1	1	2	2	2	2	1	1
5	2	2	2	2	1	1	1	1	2	2
6	2	2	2	2	1	1	2	2	1	1
7	1	1	2	2	1	2	1	2	2	2
8	1	1	2	2	1	2	2	1	2	2
9	2	2	1	1	2	1	2	2	1	2
10	2	2	1	1	2	1	2	2	2	1

Fig. 13. Regular matrix.

then it is a *global refinement*, and constitutes an alternative regular blockmodel of the matrix.

As an example, consider the 1-mode matrix in Figure 13. First, note that since every row and column of the matrix contains the same combination of values, the trivial *complete* partition (placing all rows and columns in the same block) is regular. For convenience, we refer to such matrices as *regular matrices*. A non-trivial regular blockmodel of this matrix is given by Figure 14.

Note that a local refinement of the first matrix block is the partition {{1, 2, 3, 4) {5, 6}} for both rows and columns. Further, splitting {1, 2, 3, 4} from {5, 6} in every matrix block preserves regularity, and so {{1, 2, 3, 4} {5, 6} {7, 8, 9, 10}}, shown in Figure 15, is a new global regular blockmodel. A number of further refinements, local and global, are also possible.

		A						B			
		1	2	3	4	5	6	7	8	9	10
X	1	1	1	2	2	2	2	1	1	2	2
	2	1	1	2	2	2	2	1	1	2	2
	3	2	2	1	1	2	2	2	2	1	1
	4	2	2	1	1	2	2	2	2	1	1
	5	2	2	2	2	1	1	1	1	2	2
	6	2	2	2	2	1	1	2	2	1	1
Y	7	1	1	2	2	1	2	1	2	2	2
	8	1	1	2	2	1	2	2	1	2	2
	9	2	2	1	1	2	1	2	2	1	2
	10	2	2	1	1	2	1	2	2	2	1

Fig. 14. Regular blockmodel of matrix in Figure 13.

		A				B		C			
		1	2	3	4	5	6	7	8	9	10
X	1	1	1	2	2	2	2	1	1	2	2
	2	1	1	2	2	2	2	1	1	2	2
	3	2	2	1	1	2	2	2	2	1	1
	4	2	2	1	1	2	2	2	2	1	1
Y	5	2	2	2	2	1	1	1	1	2	2
	6	2	2	2	2	1	1	2	2	1	1
Z	7	1	1	2	2	1	2	1	2	2	2
	8	1	1	2	2	1	2	2	1	2	2
	9	2	2	1	1	2	1	2	2	1	2
	10	2	2	1	1	2	1	2	2	2	1

Fig. 15. Global refinement of regular blockmodel in Figure 14.

Implied in this discussion is the fact that, in any given matrix, a number of distinct blockmodels may be regular. Borgatti and Everett (1989) have shown that, in graphs, regular equivalence forms a lattice of graph equivalences, of which structural equivalence is one member. These results are valid for 2-mode matrices as well, since any 2-mode matrix can be represented as a bipartite graph [4]. Other results on regular equivalences, such as the iterated roles of Borgatti *et al.* (1989) carry over as well.

The fact that 2-mode matrices can be represented as bipartite graphs also means that 2-mode regular blockmodels may be computed using standard network algorithms such as REGE (White 1984; MacEvoy and Freeman n.d.). Alternatively, new partitioning algorithms may be written using Theorem 2 as a basis for constructing a measure of fit to be maximized. Batagelj *et al.* (1992) have done just that for the 2-way 1-mode case. Borgatti *et al.* (1991) have implemented such an algorithm in the UCINET IV software package.

7. Image matrices for regular blockmodels

In the case of structural blockmodels, image matrices summarizing the dataset are not difficult to define. Theorem 1 guarantees that, given a perfect structural blockmodel, every value in a matrix block will be the

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[4] To construct a bipartite graph from a 2-mode matrix, let the rows correspond to one set of nodes, let the columns correspond to another set of nodes, and let the cells in the matrix correspond to the ties between the two sets of nodes. There are no ties within the node-sets.

	A	B
X	1	1,2
Y	1,2	1,2
Z	1	2

Fig. 16. Image matrix for regular blockmodel shown in Fig. 12.

same as every other value in the same block. If all the values are ones, then the corresponding cell in the image matrix is assigned a one. If every value is null, then the corresponding image value is null. The same applies to any other value as well.

Of course, in practice, matrix blocks will not be perfectly homogeneous. A block containing mostly ones may contain some zeros, or vice versa. Then, to define the image matrix, we need to specify a rule for deciding what value to assign a given block. For one-zero data, various rules have been proposed, as Faust and Wasserman (1992) discuss. One rule takes the largest value in a matrix block (i.e., if there are any ones in the matrix block, the corresponding image cell is assigned a one). Another rule takes the minimum value. Yet another rule takes the modal value (i.e., if there are more ones than zeros, assign a one; else assign a zero).

The goal that all these rules share is the assignment of a single value to summarize all the values in the corresponding matrix block. The validity of this goal hinges on the assumption (guaranteed by Theorem 1), that any heterogeneity in a matrix block is an error, indicating lack of fit, [5] and not something to be reproduced in the image matrix.

This assumption is valid for structural blockmodels, but not for regular blockmodels. [6] In general, as Figures 12, 14 and 15 demonstrate, the matrix blocks induced by regular blockmodels can have multiple values. The only constraint, as given by Theorem 2, is that every row and column of the block reproduces the same set of values as the block as whole. Consequently, it is inappropriate to seek a single value to summarize the matrix block. Rather, the summary

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[5] Panning (1982) relies on this assumption to develop his measure of blockmodel fit.

[6] The assumption applies to regular blockmodels only when the data contain only one distinct value (other than null).

	A	B	C
X	1,2	2	1,2
Y	2	1	1,2
Z	1,2	1,2	1,2

Fig. 17. Image matrix for regular blockmodel shown in Fig. 15.

consists of recording that set of distinct values that is found in every row and column of the matrix block.

Thus, the image matrix of a regular blockmodel has sets of values as its cell entries. For example, the image matrix for the regular blockmodel in Figure 12 is given in Figure 16. The image matrix for the regular blockmodel in Figure 15 is given in Figure 17.

8. Applications of 2-way 2-mode regular blockmodels

One potential area of application for 2-way 2-mode regular blockmodels is marketing research analysis. Suppose, for example, that a market researcher collects information on 300 consumers regarding brands of mustard presently found in their pantries. We organize the data as a consumer-by-brand matrix of zeros and ones, with $x(i, j) = 1$ indicating that household i possesses mustard brand j. The market researcher is typically interested in answering two questions. First, what is the structure of brands in the category? Do all brands compete equally with all others, or are there different kinds of brands which are "positioned" differently and occupy different competitive "niches" in the marketplace? Second, what is the structure of the consumers in the market? Do they all have the same needs and tastes, or are there different "segments" with different preference and purchasing patterns?

Given this type of data, the researcher typically answers the first question by what amounts to correlating columns of the data matrix and clustering, factoring or multidimensionally scaling the resulting brand-by-brand correlation matrix. In practice, other measures of column-column association might be used (such as simple co-occurrence), and some processing of the association matrix might be performed to remove marginal effects. In addition, a variety of methods other than clustering, factoring and multidimensional scaling might be used to help reveal patterns of association. Ultimately, however, all these methods reduce to finding clusters of brands purchased by the same households. These clusters are interpreted as niches, submarkets, or "positionings" as they are known in the trade. In the case of mustards, studies reliably show the existence of three major brand positionings corresponding to ordinary yellow mustard, spicy brown mustard, and fancy Dijon mustard.

The second question is answered the same way as the first. Rows of the data matrix are correlated and clustered, which is to say that sets of households are identified that have substantially similar pantry holdings. These sets are interpreted as segments. Aggregate segment-by-niche matrices show the kinds of brands that different consumer segments buy.

This analytic process is precisely equivalent to a structural blockmodeling of the household-by-brand data matrix. It is also seriously flawed. The approach hopes to find segments of consumers who are indifferent between brands of a given type, but actually assumes that the same households will purchase several (in fact, all) brands of the type of mustard they like. Only if this happens (perhaps because of brand switching within type) would we expect high correlations between brands of the same type.

A more likely scenario is that consumers usually choose only one or two brands of each type. Some consumers keep all three types of mustard on hand because they see them as useful for different occasions and purposes. Other consumers buy only one type of mustard, and others buy only two types. Different households may purchase entirely different brands of mustard, yet follow the same pattern in the sense of, say, always purchasing one yellow brand and one Dijon brand. In short, we are suggesting that while consumer segments may exist whose members have identical needs or purchasing patterns with respect to types of mustard, they should not be expected to purchase the exact same brands. Similarly, groups of similar brands may exist which serve the needs of certain consumer segments, but are not necessarily purchased by the same households. Consequently, analytic methods based on structural equivalence, such as traditional blockmodels, will fail to reveal the underlying market structure. In contrast, methods based on regular equivalence, are capable of detecting this kind of structure.

Similarly, a study of heterosexual teenage dating behavior might suppose that teenagers perceive different types of members of the opposite sex, some of which are more acceptable dates than others. A 2-mode binary data matrix recording who has ever dated whom and blocked according to types might show that boys and girls select their dates from within categories of acceptable choices. However, it is unlikely that such a blocking would be structural because that would imply that if a boy dates one girl of a certain category, then he dates all girls of that category.[7] Rather, we expect the blocking to be regular, which would imply only that certain types of boys exclusively but not exhaustively date certain kinds of girls.

As another example, consider an analysis of stock ownership which posits that there are different kinds of investors with different needs, and different kinds of stocks, with different benefits. We hypothesize that investors of a given type will share a signature pattern of investments, such as a concentration of funds on high income stocks with little diversification. Similarly, we assume stocks fall into categories attracting different types of investors. However, we don't necessarily expect two investors of the same type to invest in the same stocks: it is sufficient that they invest in the same kinds of stocks. Likewise, we do not expect two stocks of the same type to attract exactly the same investors, merely the same type.

Two-mode regular blockmodels are particularly useful for modeling matrices meeting the following general description. First, row entities choose column entities (i.e., have a certain relationship with), and vice versa. Second, there are different types of row entities, and different types of column entities. Third, the different underlying types of row entities are distinguishable by the fact that they choose different types of column entities. Similarly, different types of column entities are distinguished by choosing different combinations of types of row entities. Fourth, given that a row entity belongs to an underlying class that chooses column entities of a given type, the choice or choices of particular column entities is arbitrary. Similarly, column entities choose

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[7] It should be noted that the issue is not that data are never perfect and therefore no boy is likely to date absolutely all the girls he is supposed to and none of the girls he is not supposed to. The issue is that the ideal images at which structural blockmodels are aimed are, in this case, inappropriate on logical grounds. Even if approximate measures of structural equivalence or best-fitting structural blockmodels are identified, the data will never be fit by the model: it is the wrong model for this sort of data.

row entities randomly [8] within class of row entity. The first three conditions describe data appropriate for all blockmodels, including structural blockmodels. The fourth condition distinguishes data conforming to the structural model from data conforming to the regular model.

9. Applications of 3-way regular blockmodels

Consider a criminal-by-crime-by-victim matrix, which we regard as 3-mode. Assume the criminals consist of both individuals and groups, as do the victims. The crimes include burglary, larceny, rape, assault, homicide, and robbery. A regular blockmodel would simultaneously partition criminals, crimes, and victims such that equivalent criminals commit the same kinds of crimes against the same types of victims. Similarly, equivalent crimes are those which tend to connect the same classes of criminals with the same types of victims. Such an analysis might find, as Stark (1989) suggests (see also Stark *et al.* 1980; Crutchfield *et al.* 1983), that there are two types of crimes: intentional and impulsive. The intentional crimes (burglary, larceny, rape) are "rational", pre-planned and have rates well-predicted by societal variables such as population turnover rates and proportion of churchgoers. In contrast, the impulsive crimes (assault, homicide, robbery), are "irrational", situational, and better explained by psychological variables. The analysis might further find that there are three kinds of criminals: adult professionals, teenagers, and "hot-heads". The three types are characterized by the kinds of crimes they commit, with the professionals committing only intentional crimes, the teenagers committing both kinds, and the hot-heads committing only impulsive crimes. Similarly, the victims can be classified according to the kinds of crimes and criminals they fall prey to.

In this example, crimes act essentially as binary relations which link pairs of criminals and victims. Thus, one application of 3-way regular blockmodels is to extend the notion of regular equivalence to apply not only to nodes but also to relations in a network. Consider, for example, a 3-way, 2-mode actor-by-actor-by-relation matrix such as

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[8] In this context, the term "random" means simply that choices are made according to criteria independent of and irrelevent to the blockmodel.

the well-known Sampson (1968) data. A regular blockmodel of this data partitions actors into blocks whose members are connected to equivalent others on *equivalent* relations. This contrasts with the standard approach to handling multiple relations, which is to require regular equivalence to hold across each relation simultaneously. In this approach, two actors in a network are considered equivalent if they are connected to equivalent others on the *same* relations. In the context of a 3-way blockmodel, this amounts to restricting the relation partition to be the identity partition in which each relation is considered unique and placed in its own class. At the extreme opposite of this approach is the possibility of constraining the relation partition to be the complete partition in which all relations are considered equivalent and are placed in the same class. This yields a blockmodel in which equivalent actors are only required to have a tie with equivalent others on any relation. This is equivalent to finding a regular equivalence on the simple graph formed by the union of all relations.

Another application of 3-way regular blockmodels arises in network analysis when the unit of observation is triads rather than dyads. For example, suppose we observe conversations at a party. As Seidman (1981) has noted, recording a conversation among three people as three separate pairwise interactions fails to capture the essential piece of information, which is that all three were in the same conversation, not three separate ones. Figure 18 gives the triads observed at a hypothetical party for six guests. To seek a 1-mode 3-way regular blockmodel of this data is to ask the following question: is there a role structure among guests that patterns the kinds of triads that can occur? Although not obvious by simple inspection, the answer in this case is yes. The actors form three blocks: A = {1, 2}, B = {3, 4}, and C = {5, 6}. Members of the same block engage in conversations with the same combination of types of other actors. The blocking of actors,

```
{1,2,5}
{1,2,6}
{1,3,6}
{1,4,6}
{1,5,6}
{2,3,5}
{2,4,5}
{2,5,6}
```

Fig. 18. Triads observed at a hypothetical party.

```
        {1,3,6}
ABC     {1,4,6}
        {2,3,5}
        {2,4,5}

        {1,2,5}
 AC     {1,2,6}
        {1,5,6}
        {2,5,6}
```

Fig. 19. Regular blocking of triads shown in Figure 18.

as always, induces a blocking of ties, which in this case represent conversations. As Figure 19 indicates, only two kinds of conversations are observed at this party. One kind, labeled ABC, involves a representative of each type of actor. For example, one conversation involves actors 1, 3 and 6; Another involves 2, 4, 5. Even though these two conversations involve none of the same actors, they are equivalent by virtue of involving the same types of actors. The other kind of conversation, labeled AC, involves only guests of type A and type C. In fact, all possible triads involving only these two types of actors are observed.

Relationships among triads may be derived as well as directly observed. For example, we might compute the relation "i is indirectly connected to j via a two-step path through k". Regularly equivalent actors are those which are indirectly linked to equivalent others by equivalent brokers. A 4-way 2-mode blockmodel can be used to model the relation "i is indirectly connected to j via a k-step path through l (among others)."

Regular blockmodels may also be used to study patterns in sequences, such as words in a sentence. For example, consider the set of 3-word sentences given in Figure 20. The data may be represented as a 3-way, 1-mode word-by-word-by-word matrix in which $x(i, j, k) = 1$ if there exists a sentence consisting of word i followed by word j

```
Dogs chase cats.
Cats like girls.
Girls like boys.
Boys like dogs.
```

Fig. 20. A set of 3-word sentences.

followed by word k. A regular blockmodel of this data consists of the following blocking: {{boys, cats, dogs, girls}, {chase, like}}. The image matrix consists of a 3-way matrix which is empty except for cell (1, 2, 1), which is one. If we label the first block "nouns" and the second block "verbs", then the image matrix tells us that the only observed sentence structure is of the form noun → verb → noun. Thus, regular blockmodels can be used to generate models of some syntactically organized phenomena. For example, Propp (1968) and Colby (1973) have suggested that sequences of events in myths and folktales of a given culture and genre conform to simple grammatical rules that govern what kind of events can occur at a given point in a story. Thus, a given myth (or "sentence") is a single datapoint or cell in a large multiway matrix, where each way corresponds to a position in the sentence. The image matrix from a regular blockmodel of such a matrix gives the set of sentence types that are permitted by the grammar.

Other phenomena that might be analyzed this way include career trajectories, political events, and behavioral scripts (Nowakowska 1973; Schank and Abelson 1977; Skvoretz 1984). A technical application along these lines is the search for semigroup homomorphisms. To operationalize the task as a regular blockmodeling problem, we represent the semigroup's multiplication table as a binary 3-way, 1-mode matrix such that $x(i, j, k) = 1$ iff postmultiplying element i by element j yields element k. The resulting blockmodel will identify classes of elements such that if elements e_1, e_2 are in the same class, and f_1, f_2 are in the same class, then all products $e_i f_j$ will be members of the same class. Thus, the image of the blockmodel gives the multiplication table for a (regular) semigroup homomorphism. It is interesting to note that in the case of groups (rather than semigroups), the multiplication table is always regular.

As with regular blockmodels of 2-way matrices, regular blockmodels of multiway matrices are used to describe regularities that occur at the level of type rather than individual. Viewed from the point of view of matrix blocks, multiway regular blockmodels identify sets of p-tuples (matrix cells) that are equivalent because they are combinations of the same types of elements. Equivalent p-tuples are like investment portfolios which, while not containing precisely the same securities, include representatives of all the same types. They are like distinct representative samples drawn from the same population.

10. Conclusion

In this paper, we extend the notion of blockmodeling in two ways. First, we show how blockmodels can be adapted to incorporate regular equivalence as an alternative to structural equivalence as a basis for blocking. Second, we show how to apply blockmodels to multiway, multimode matrices. Thus, regular blockmodels can be used not only to analyze networks analysis, but to find structure in many different kinds of datasets. In the process, we also shift the focus of attention away from the blocking of actors (or, more generally, ways) and toward the blocking of ties (cells). Both theorems concern the characteristics of classes of matrix cells formed by structural and regular blockmodels. Focusing on equivalent cells makes it easier to conceptualize blockmodels of multiway matrices. It is also very appropriate for structural analysis, inasmuch as the objects of analysis are not individuals but relationships among them.

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