Journal of Classification 12:57-71 (1995)

Fitting an Extended INDSCAL Model to Three-Way Proximity Data

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Abstract: The INDSCAL individual differences scaling model is extended by assuming dimensions specific to each stimulus or other object, as well as dimensions common to all stimuli or objects. An ''alternating maximum likelihood'' procedure is used to seek maximum likelihood estimates of all parameters of this EXSCAL (Extended INDSCAL) model, including parameters of monotone splines assumed in a ''quasi-nonmetric'' approach. The rationale for and numerical details of this approach are described and discussed, and the resulting EXSCAL method is illustrated on some data on perception of musical timbres.

Keywords: Weighted Euclidean model; INDSCAL; Multidimensional scaling; Specificities, Monotone splines.

1. Introduction

We consider a general three-way multidimensional scaling (MDS) model in which distances are of the form:

We acknowledge the referees for their careful and conscientious attention to this manuscript.

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$$d_{ijk} = \left[\sum_{t=1}^{T} w_{kt} (x_{it} - x_{jt})^2 + s_{ik} + s_{jk} \right]^{1/2} \qquad (1)$$

where x_{it} ($i = 1,2, \ldots ,I$, $t = 1,2, \ldots ,T$) is the coordinate of the i-th stimulus or other object on the t-th common dimension, w_{kt} ($k = 1,2, \ldots ,K$) is the weight of the k-th subject or other source of data on the t-th common dimension, while s_{ik} is the k-th source's ''specificity'' for the i-th stimulus.

The equation for source distances given in (1) is a generalization of that for distances between objects in the two-way version of this model discussed by Winsberg and Carroll (1989b), and of the restricted three-way version discussed in Winsberg and Carroll (1989a) while the method for fitting the corresponding model generalizes the method discussed in the latter paper. The three-way distances generalize the two-way distances by inclusion of the source weights, w_{kt}, and by adding a source subscript (k) to the specificities s_i and s_j. Since the first term of the resulting three-way weighted distance formula, based on the common dimensions (x_{it}), is identical to that in the INDSCAL model (Carroll and Chang 1970), we call this development an ''Extended INDSCAL model,'' extended to include the specificities s_{ik} and s_{jk}. The Extended INDSCAL model can be viewed as a special case of the INDSCAL model, with many more dimensions than the common dimensionality, T: with up to I more dimensions, but with very strong constraints on these additional ''specific'' dimensions; namely, that a specific dimension for object i has non-zero coordinates only for that i-th object. The specificity, s_{ik}, is the product of the weight of source k for that specific dimension and the square of that one non-zero coordinate value for the dimension (for object i). There could be, in principle, *more* than one specific dimension for each object, while the specificity s_{ik} would be defined as the source weight multiplied by the sum of squares of those non-zero coordinate values, but this formulation is mathematically indistinguishable from the case of exactly one specific dimension per object. One could also assume a different specific dimension for each source/object combination, but this formulation to is also mathematically indistinguishable from the present formulation; thus parsimony dictates assuming only one specific dimension per object.

It is possible, of course, that there is no specific dimension for a particular object. This situation is equivalent, however, to a specific dimension whose ''non-zero'' coordinate is in fact equal to zero, so that the specificity s_{ik} is also zero for all k. The specificity for a particular source, k, of the i-th object can also result from the source's weight for that specific dimension being zero.

While the assumption of more dimensions than objects may seem at first to lead to a model not fully identified, the very strong constraints on these specific dimensions obviate this ostensible problem. Recall also that the three-way INDSCAL model allows more dimensions than objects *or* sources (see, e.g., Kruskal 1976), even without these strong constraints on the dimensions' structure. While it has not yet been established mathematically how many extra dimensions can be included without loss of identifiability of the model parameters, we believe that the IK extra parameters corresponding to specificities do not lead to unidentifiability problems in general. Winsberg and Carroll (1988) demonstrated that the parameters are identified in the two-way case (which, of course, is a special case of the three-way model proposed here, with $K = 1$), so we see no reason why if $K \geq 2$, the parameters should not be identified. In the present paper, however, we shall not consider fitting the general extended INDSCAL model, whose distances are defined in equation (1), but rather a restricted case of this model to be described below. We plan to implement the completely general model, and thus test our conjecture that it can be fit uniquely (at least with sufficiently small values of T) in subsequent research. Specificities are often necessary to include in a spatial model when the data exhibit centrality (see Winsberg and Carroll (1989b), which quantifies (see Pruzansky, Tversky and Carroll 1982) the degree to which there exists an object that tends to be the "nearest neighbor," or at least a "near neighbor," (i.e., one that is very close, relative to distance of other objects, albeit not necessarily the *closest*) to all other objects. Such source-dependent parameters as the weights are useful for finding potentially meaningful psychological dimensions (see Carroll and Chang 1970).

While in the general model proposed above, each subject or other source of data is assumed to have its own unique set of specificities, a more restricted model would assume that each subject has the same pattern of weights for the object-specific dimensions, with the sources differing only in the overall salience of these specific dimensions relative to that of the T common dimensions. If we assume the specificity of object i to be s_i for a source with unit salience weight, while the salience weight is u_k for the specific dimensions as a whole for source k, this formulation leads to the specificity of source k on object i, s_{ik}, being of the form:

$$s_{ik} = u_k s_i \,. \tag{2}$$

This formulation leads to the restricted distance equation:

$$d_{ijk} = \left[\sum_{t=1}^{T} w_{kt} (x_{it} - x_{jt})^2 + u_k (s_i + s_j) \right]^{1/2} . \tag{3}$$

This restricted model involves only $I + K$ additional parameters, rather than the IK additional parameters required by the fully general extended INDSCAL model whose distances are defined in equation (1).

The data for three-way MDS are generally a three-way ($I \times I \times K$) array of proximities (similarities or dissimilarities). We shall assume henceforth that these values are dissimilarities, and denote the general entry in this data array δ_{ijk} (the dissimilarity between objects i and j for source k). While we may often assume the data are measured on ratio or interval scales, so that a metric procedure, such as that described by De Soete, Carroll and Chaturvedi (1993) is feasible, in many cases we can only plausibly assume ordinal scale data. We would then like to have a procedure for *fully nonmetric* fitting of the model, assuming ideally, a completely general monotonic function relating data and distances. We define "fully nonmetric" fitting via two conditions: (a) A fully nonmetric procedure yields solutions which are invariant under strictly monotonic transformations of the data (in the case of matrix conditional data, separately within each matrix, or source, of data); and (b) If one or more solutions exist such that the rank order of distances agrees perfectly with the rank order of the data, in the sense that at least a weak monotonic function (but *not* the limiting case of the "trivial" monotonic function constant throughout its range, which preserves *no* ordinal information) relates the data to distances, so that the ordinal properties of the data are fully accounted for, the procedure will produce *one* of these solutions, subject to the usual caveats of possible "locally optimal" solutions. We use the term "quasi-nonmetric" to denote an approach which closely approximates but does not precisely satisfy this strict definition. We show below that there are theoretical and methodological obstacles to fully nonmetric fitting — most seriously — a problem of "degenerate" solutions, in the case of the Extended INDSCAL model.

2. The Degeneracy Problem

Let us first assume that, as in the approach to nonmetric MDS exemplified by Kruskal's (1964a, 1964b) STRESS-based algorithm, we attempt to transform the data (δ's) monotonically to match fitted distances (d's). As detailed by Carroll (1988), this strategy leads to a serious degeneracy problem in fitting the two-way version of the present model, as well as for a wider class of models, including tree structure models (De Soete 1988), ADCLUS/MAPCLUS/INDCLUS (Arabie and Carroll 1980; Carroll and Arabie 1983; Shepard and Arabie 1979), and other "discrete" models (Carroll 1976; Carroll and Pruzansky 1980; DeSarbo 1982).

Since we treat the data as matrix (subject or source) conditional (Takane, Young and de Leeuw 1977), we assume a separate monotonic function for each data source. Assume we are first dealing with the special case of $K = 1$, corresponding to the case of "two-way" proximity data. We define a degenerate solution as one that asymptotically predicts a vanishingly small proportion of the (ordinal) information in the original data within each matrix. The number of potential pairwise ordinal relations among the $\begin{bmatrix} I \\ 2 \end{bmatrix} = \frac{I(I-1)}{2} = N$ distances between I objects is $\begin{bmatrix} N \\ 2 \end{bmatrix} = \frac{(I+1)(I)(I-1)(I-2)}{8} \rightarrow \frac{I^4}{8}$ as $I \rightarrow \infty$. Thus any solution asymptotically allowing prediction of no more than order I^3 ordinal relations would be degenerate.

In the two-way case, a seemingly "perfect" (in the sense of accounting fully for the ordinal properties of the data), but degenerate solution can always be found when the smallest dissimilarity (largest similarity) value is not tied. This solution predicts only the identity of that (unique) pair having this smallest dissimilarity value, and thus only the ordering of the $N - 1$ distance pairs containing the corresponding (smallest) distance as a member. Since $N - 1 = \frac{I(I-1)-2}{2} \rightarrow \frac{I^2}{2}$ = order I^2, by the above definition this configuration is clearly degenerate.

This "degenerate" solution can be obtained assuming only a *single* common dimension. On this single dimension the two objects corresponding to the smallest dissimilarity coincide in a single point (have an identical coordinate value) while the remaining $I - 2$ points coalesce into a second distinct point. Let i' and j' be the subscripts of the two points corresponding to this smallest dissimilarity; thus we are assuming $\delta_{i'j'} < \delta_{ij}$ for $(i,j) \neq (i',j')$ (where (i,j) denotes an unordered pair of subscripts). Now, the distances, d^*, on this specially constructed "common dimension" are of the form:

$$\begin{aligned} &d^*_{i'j'} = 0, & \\ &d^*_{i'j} = c > 0 \qquad && \text{for } j \neq j', \\ &d^*_{ij'} = c && \text{for } i \neq i' \\ &d^*_{ij} = 0 && \text{for } i \neq i' \text{ and } j \neq j' \end{aligned}$$

We now assume specificities for this two-way extended Euclidean model of the form:

$$\begin{aligned} &s_{i'} = s_{j'} = 0 \\ &s_i = c \;\; \text{for } i \neq i' \text{ or } j'. \end{aligned}$$

Then we have *extended* Euclidean distances $d_{ij} = d^*_{ij} + s_i + s_j$ of the form:

$$d_{i'j'} = d^*_{i'j'} + s_{i'} + s_{j'} = 0 + 0 + 0 = 0\,,$$
$$d_{i'j} = d^*_{i'j} + s_{i'} + s_j = c + 0 + c = 2c,\ \text{ for } j \neq j'\,,$$
$$d_{ij'} = d^*_{ij'} + s_i + s_{j'} = c + c + 0 = 2c,\ \text{ for } i \neq 1'\,,$$
$$d_{ij} = d^*_{ij} + s_i + s_j = 0 + c + c = 2c,\ \text{ for } i \neq i',\, j \neq j'\,.$$

Thus $d_{i'j'} = 0$, but all other distances are tied at the single value $2c > 0$. Any approach that would treat a solution in which recovered distances are a nontrivial but possibly weak monotonic function of the data as a "perfect" solution would be degenerate in the sense defined above. Examples of this type of approach are: Kruskal's MDSCAL (Kruskal and Carmone 1969) or KYST (Kruskal, Young and Seery 1977); this remark is valid for STRESSFORM 1 as well as STRESSFORM 2 (see Kruskal and Carroll 1969). This degeneracy could also occur for a large class of other actual or potential nonmetric MDS algorithms optimizing such "stress-like" objective functions as SSTRESS, in Takane, Young and de Leeuw's (1977) ALSCAL, or the "rank image principle" in Guttman's (1968) Smallest Space Analysis. While such a precise degeneracy may not obtain in the case in which there are ties in the data involving the smallest dissimilarity value, we strongly suspect that even in such "untied" data there are at least quasi-degeneracies that approximate this (asymptotically) totally uninformative fully degenerate situation.

In the case of the generalization to $K \geq 2$ (i.e., the general extended INDSCAL model), if we assume no ties for the smallest dissimilarity for each source (and treat the data as matrix conditional) the degenerate solution described generalizes without any problem if we assume as many common dimensions (T) as sources, allowing a "common" dimension for each source, with that source's most similar object pair joined into one point on that dimension, and the other $I - 2$ objects into a second. If the same object pair is maximally similar for two or more sources, a solution with fewer dimensions would produce a degenerate solution. As in the two-way case, a "quasi-degenerate" solution could be expected in cases involving ties.

Now we replace such "fully nonmetric" fitting with fitting of a class of continuous functions constrained to be monotonic; e.g., continuous monotone splines (Winsberg and Ramsay 1980). So long as this function is sufficiently general to allow it to approach the one-step monotone function (weakly monotone everywhere except where the step occurs), then the same class of degeneracies is still possible.

Now, let us consider the case in which we reverse the usual definition of independent and dependent variables and seek a monotonic function of the distances that optimally predicts the data. The problem, if we allow a totally general monotone function, is that such a function is typically a step function, for which derivatives either do not exist or are zero where they do exist. Since essentially all numerical procedures for fitting parameters of continuous functions require at least first (and sometimes second) derivatives of the objective function to exist, this property makes this approach numerically infeasible. However, a "quasi-nonmetric" approach is feasible here, using, for example, continuous monotone splines which can be constrained to have derivatives that exist everywhere. It is, therefore, this particular "quasi-nonmetric" approach characterized by using monotone splines that we shall use to fit the extended INDSCAL model described above, restricting our attention to continuous monotone splines with first derivatives that exist everywhere. (The step functions used by Kruskal [1964b] are indeed first order splines with knots at each data point, but this class of splines does not meet our criteria.)

Transformation of the model values, as opposed to transformation of the data values, can be related to the process by which the data are assumed to be generated. This relationship is discussed in Winsberg and Carroll (1989b) and Winsberg (1988). Also, from a practical statistical standpoint, use of such monotonically constrained continuous functions allows tractable interpolation, thus allowing reconstruction of expected similarity or dissimilarity corresponding to missing data values.

3. I-Splines

Monotone splines denoted I-splines, first developed by Winsberg and Ramsay (1980, 1983), are a class of functions which are extremely flexible but require only a few parameters for representation. They are easy to compute, integrate, and differentiate; the last is an essential property for the current application. A full presentation and description of the properties of I-splines can be found in Winsberg (1988) and Winsberg and Ramsay (1983). I-splines are preferred to B-splines in this context since I-splines may be constrained to be monotone simply by putting positivity constraints on the parameters, and the relationship between distance and dissimilarity should be expressed as a monotone function.

4. Parameter Estimation

The parameters in the restricted model belong to one of three sets: the spatial parameters including both the common and specific dimensions x_{ip} and s_i $(i = 1, \ldots, I; p = 1, \ldots, P)$, the transformation parameters c_q, $(q = 1, \ldots, Q)$, and the weights w_{kp} $(k = 1, \ldots, K; p = 1, \ldots, P)$, and u_k $(k = 1, \ldots, K)$. The algorithm we employ is a maximum likelihood procedure and alternates among (a) maximizing the log likelihood with respect to the spatial parameters, conditional on the weights and transformation parameters, (b) maximizing the log likelihood with respect to the transformation parameters conditional on the spatial parameters and the weights, and (c) maximizing the log likelihood with respect to the weights (if they are not set equal to unity in the model) conditional on the spatial parameters and the transformation parameters.

Step Zero of our procedure starts with a linear transformation and weights of unity and finds the configuration that maximizes the log likelihood. Each following step of the algorithm has three substeps: In Substep One, conditional on the configuration and weights, we find the transformation that maximizes the log likelihood. In Substep Two, conditional on the configuration and the transformation, we find the weights that maximize the log likelihood. (If the model constrains the weights to be unity, this step is omitted, and the algorithm will go directly to Substep Three from Substep One.) In Substep Three, conditional on the transformation and the weights, we find the spatial configuration that maximizes the log likelihood. Substep Three concludes the alternation process. After each alternation process or step the log likelihood is compared to the value obtained in the previous process or step. The algorithm stops if either improvement in the log likelihood has converged to a suitable value or the log likelihood does not increase. If the conditions for stopping the algorithm are not met, we proceed to the next step which consists of the three substeps outlined above. In each substep we use a modified Fisher scoring method as described below. The substep is considered to have converged when the improvement in the function is less than 0.001 of its value.

Constraints are imposed to identify the model. Centering does not affect the model distances. Moreover, if we use a special case of this model constraining the weights to be unity, rotation does not affect the model distances either. The monotonicity condition for the transformation requires that the transformation parameters be positive. In addition we constrain the specificities and the weights to be non-negative, to keep the distances non-negative and to guarantee satisfaction of the metric axioms. The non-negativity constraints on these sets of parameters are obtained using a technique in which a subset of the parameters corresponding to active constraints are held fixed at each iteration (see Gill, Murray and Wright 1981). An active constraint occurs when a parameter has become zero and the corresponding gradient element is negative. At each iteration, if any of the constraints are active, the reduced gradient, $\mathbf{g}_{red}$, and corresponding reduced expected Hessian, $\mathbf{H}_{red}$, are used in place of the entire gradient, $\mathbf{g}$, and expected Hessian, $\mathbf{H}$, where $\mathbf{g}_{red}$ and $\mathbf{H}_{red}$ are the portions of $\mathbf{g}$ and $\mathbf{H}$ corresponding to inactive constraints respectively. That is, the rows of $\mathbf{g}$ corresponding to active constraints and these same rows and columns of $\mathbf{H}$ are eliminated from $\mathbf{g}$ and $\mathbf{H}$

respectively, and the active parameters are held fixed during the iteration. After each iteration the set of active constraints is redetermined. Alternatively, barrier functions could be used to impose the non-negativity constraints. In some unpublished work, one of the authors has compared both of these techniques and found that they work equally well.

We use a modified Fisher scoring algorithm using $-\mathbf{H}_{red}^{+}\mathbf{g}_{red}$ for the search direction where $\mathbf{H}_{red}^{+}$ is the Moore-Penrose inverse of the information matrix (expected Hessian) for the set of parameters being estimated. This use of the Moore-Penrose inverse has been described by Ramsay (1980). Using the Moore-Penrose inverse of the expected Hessian allows one to estimate the appropriate number of free parameters which are identifiable given the model, and then (say) center the data a posteriori. We were also able to check that all of the parameters actually being estimated were properly identified by examining the eigenvalues of this matrix. See Winsberg and Carroll (1989b) for a more complete discussion of this point. Since we have maximum likelihood estimates, we may compare some of the models according to their quality of fit per parameter. Some parameters may lie on the boundary of the constraint surface for the model being fitted, however, and then routine application of standard asymptotic procedures is not possible. Of course, when estimating individual weights or transformations, asymptotic procedures are not available, since the number of "nuisance" parameters increases as the number of observations increases. Maximum likelihood estimators are inconsistent estimators in the presence of nuisance parameters (see Neyman and Scott 1948; Kiefer and Wolfowitz 1956). The use of AIC and BIC (Aikake 1974; Schwartz 1978) statistics to select among spatial models is justified when they do not contain nuisance parameters relating to sources. Even in the case of the unweighted model, such nuisance parameters are needed for fitting transformations. For individual differences models, where asymptotic theory does not justify using AIC or BIC, heuristic methods must be used to select the model. One rule of thumb is that each dimension be interpretable. Another rule of thumb is to require at least as stringent a criterion as would be applicable if the asymptotic theory were valid; in particular that the AIC statistic for the chosen model should be at least a minimum. This last statement is equivalent to requiring that the log likelihood should increase by at least the additional number of parameters.

5. Example

The ability of this procedure to recover parameters under various conditions for the restricted model (no specificities and all weights equal to unity; varying weights) was previously investigated by Winsberg and Carroll (1989a). We will now proceed to discuss a real example. Twenty-one different musical timbres were selected and generated at IRCAM (l'Insitut de Recherche et Coordination Acoustique Musique) using the frequency modulation technique of Wessel, Bristow and Settel (1987). Some of these timbres were designed to simulate traditional instruments (e.g., the trombone, the horn, the oboe, the harpsichord, and the guitar) and some were synthesized to have some of the qualities of each of two traditional instruments; that is, they were hybrid timbres (e.g., the striano, string and piano, and the guitarnet, guitar and clarinet). One in particular was synthesized to have some special qualities in addition to its hybrid nature (the vibrone, a hybrid of the vibraphone and trombone). Other timbres as well were thought to have special qualities. In fact this experiment was conducted in part to test whether attributes specific to individual timbres exist in addition to the dimensions common to all timbres which had been found in previous studies (Grey 1975; Wessel 1973). Thus, this model was appealing for the analysis of these data. These timbres are listed in Table 1. Dissimilarity data for nine subjects, collected by Krumhansl (1989), were then analyzed using the Winsberg-Carroll algorithm.

Since the use of AIC and BIC statistics is inappropriate in this context, we have used the heuristic criterion requiring each retained dimension to be interpretable for model selection. Table 2 shows the log likelihood obtained for each model tested, the degrees of freedom (including those for nuisance parameters), and the AIC statistics based on this number of degrees of freedom. These AIC statistics should not be used for model selection, as indicated earlier, but only as very rough indicators of fit to be used together with criteria based on interpretability, parsimony, and other heuristic indicators of dimensionality and appropriateness of other parameters.

The objects with large specificities were generally designed or recognized by musicians to possess some unique attributes indeed. The weights, while adding only slightly to the fit, were of utmost importance in recovering an interpretable orientation for the three-dimensional common space. The uniformly weighted solution was rotated interactively without success to try to recover interpretable dimensions. On the other hand, the weighted Euclidean model provided easily interpretable dimensions. Thus, we rotated the Euclidean solution for the uniformly weighted model to fit the solution for the weighted Euclidean model, as shown in Figures 1 and 2.

The first common dimension represents the rapidity of the attack, with the stimuli of fast attack (e.g., the harpsichord and the guitar) at the upper end of the dimension, and stimuli of slow attack (e.g., the horn, oboe, and the bassoon) at the lower end. The second dimension, which we denote spectral flow, seems to correspond to a nonlinear temporal evolution of the spectral components. This dimension is evidenced at the high end by the brass instruments; e.g., the trumpet. Indeed for many years the trumpet and other brass

Table 1:

Stimuli Used in Timbre Study

Name of Stimulus	Instrumental Composition
horn	French horn
trumpet	trumpet
trombone	trombone
harp	harp
trumpar	trumpet & guitar
oboleste	oboe & celeste
vibraphone	vibraphone
striano	string & piano
sampiano	sampled piano
harpsichord	harpsichord
cor anglais	English horn (tenor oboe)
oboe	oboe
bassoon	bassoon
clarinet	clarinet
vibrone	vibraphone & trombone
obochord	oboe & harpsichord
pianobow	piano (bowed)
guitar	guitar
string	string
piano	piano
guitar net	guitar & clarinet

Table 2:

Summary of Solutions for Timbre Data

wts	specificities	dim	spl	LOGL	df	AIC
no	no	2	no	-2400.5	56	4913.0
no	no	3	no	-2250.9	74	4649.8
no	no	4	no	-2118.3	91	4418.6
no	no	5	no	-2085.3	107	4360.6
no	yes	3	no	-2215.8	95	4621.6
yes	yes	3	yes	-2202.9	143	4691.8

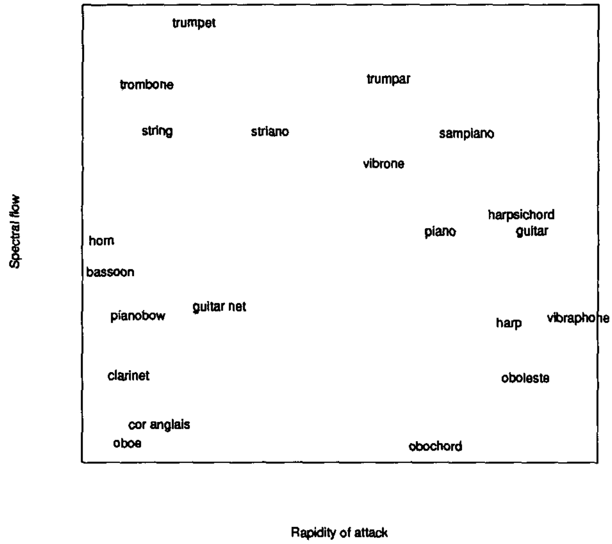

Figure 1: Dimension 1 (rapidity of attack) and Dimension 2 (spectral flow) of the stimulus space of the rotated timbre data solution.

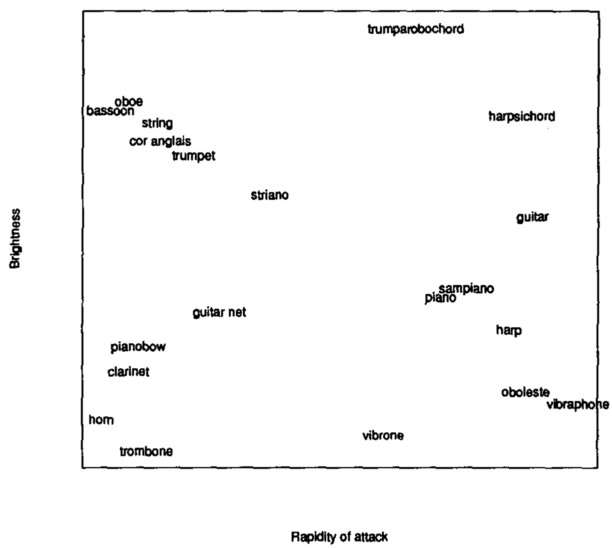

Figure 2: Dimension 1 (rapidity of attack) and Dimension 3 (brightness) of the stimulus space of the rotated timbre data solution.

instruments could not be adequately synthesized. It was only when some researchers (see Risset 1986) did not limit themselves by considering the spectrum as invariant, that the instruments such as the trumpet could be synthesized; so this dimension is indeed important. The third dimension represents brightness, or the center of the Fourier spectrum, with instruments like the oboe and harpsichord at the upper end, and the horn and the trombone at the lower end. Moreover, these dimensions are similar to those found in the scaling study conducted by Grey (1975). The hybrid timbres generally occupied positions between those of the two timbres used for their synthesis. Stimuli of higher specificity, musically speaking, like the vibrone, did indeed yield high specificities in our analysis. The solution also indicated high specificities for other timbres, namely the clarinet, the harpsichord, the piano-bow, the guitarnet, and the harp. The offset of the harpsichord is distinctive, and the clarinet is the sole instrument that does not have even harmonics. These findings then indicate that timbres may have unique characteristics, as well as a small number of common dimensions. The spline transformations (integrated linear splines with one knot plus an additive constant, resulting in four parameters per fitted spline function) did not improve the model fit appreciably. They indicate only a mild variation from subject to subject, and in most cases, mild departures from linearity; a linear solution was therefore retained.

Both the other additional features — the INDSCAL-like weights and *particularly* the new feature of specificities — were of the utmost importance in recovering interpretable dimensions for this set of data. Indeed without these features the recovered dimensions were not interpretable, and the specificities are of special interest. This analysis has proved useful for understanding perceptual aspects of timbre space. British composer James Dillon used these aspects of timbre space, based on our analysis, to generate sounds for his composition, *Introitus*, for twelve strings, tape, and electronics (Dillon 1990).

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