# THREE-WAY MULTIVARIATE CONJOINT ANALYSIS*

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Three-Way Multivariate Conjoint Analysis is developed as an extension of traditional metric conjoint analysis allowing one to examine several dependent variables simultaneously, as well as individual differences in response. Four nested models are developed to examine the effects of the experimental design, the dependent variables, and individual differences. An illustration concerning the relationship of product characteristics to the importance of various decision-making criteria for industrial purchasing is provided. Finally, extensions of the model(s) to other marketing applications and nonmetric analyses are discussed.

**(Conjoint Analysis; Three-Way Multidimensional Scaling; Constrained Preference Analysis)**

## 1. Introduction

Traditional metric conjoint analysis typically entails gathering preferences for various stimulus profiles and then decomposing the preference scores via regression (or other methods) to obtain the utilities or part-worths for the various levels of the attributes of the stimuli (Green and Rao, 1971). The purpose of this paper is to develop an extension of metric conjoint analysis to account for the effects of *both* multiple dependent variables *and* individual differences in the model. Individual differences have typically been examined via separate analyses for each individual (or segment), or have been ignored when aggregate analysis is performed. In the model presented here, they are allowed to directly impact on the importances of the attributes, allowing one

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to quantify their impact and interrelationships. The model also allows for the inclusion of multiple dependent variables, much as canonical correlation extends regression. The models proposed here allow for a graphical representation of individual differences, part-worths, and dependent variable effects.

The paper begins by presenting a brief review of traditional conjoint analysis and its limitations. Next, Three-Way Multivariate Conjoint Anlaysis is described and four nested models developed with algorithms for parameter estimation. A sample illustration concerning the relationship of product characteristics to the importance of various decision-making criteria for industrial purchasing is provided. Finally, extensions of the model to other marketing applications and nonmetric analyses are discussed.

## II. Conjoint Analysis

### A. *A Brief Review of Traditional Conjoint Analysis*

Green and Srinivasan (1978) trace the historical development of conjoint analysis and provide a comprehensive summary of the major contributions to this field. According to these authors, one popular approach utilized today consists of the part-worth function model, full-profile, fractional factorial design, metric rating scales, any type of stimuli presentation, and multiple regression combination. Here, the analyst initially selects the relevant stimulus attributes to be considered, specifies their various levels, and considers any interactions that seem appropriate to estimate. After this series of steps, the analyst constructs a fractional factorial design (Green, 1974) to estimate the desired parameters. The experimental profile combinations from this design are then presented (occasionally pictorially) to subjects who are asked to rate each full-profile combination in terms of overall desirability or preference. The data can then be pooled or averaged over subjects and multiple regression can be used to estimate the part-worths, where the experimental design (converted to dummy variables) becomes the set of independent variables (Green and Tull, 1978). Alternatively, analysis can be performed at the individual level and a choice simulator aggregates predictions.

### B. *Limitations of This Approach*

There are two major limitations with this form of conjoint analysis:

(1) The methodologies usually deal with a single dependent variable or, if multiple dependent variables are used, deal with each dependent variable separately. A notable exception is the path analyses employed by Holbrook (1981);

(2) The research must choose among two extremes in handling individual differences:

(a) aggregate individuals and hence mask individual differences, or

(b) analyze data separately at the individual or segment level.

1. *Univariate vs. Multivariate Analysis.* Many situations exist where one is concerned with more than one dependent variable. As one example, consider an analyst attempting to assess the impact of marketing strategy (advertising budget, price, etc.) on various objectives such as sales, market share, and

profitability. Or, an automobile manufacturer might be interested in the impact of such attributes as color, miles per gallon, and front vs. rear wheel drive, etc., on perceptions of a car's sportiness, roominess, appropriateness for the respondent's life-style, as well as overall preference. While one could perform a multiple regression analysis *separately* on each dependent variable (or use multivariate regression), there is no technique available to perform conjoint analysis and simultaneously examine the *interrelationships* among the many dependent variables of interest.

2. *Individual/Segment Differences.* Often, one pools or averages the data over subjects before performing multiple regression. However, this procedure can render quite misleading results. As a simple intuitive example, consider just two subjects. Clearly if their preference ratings are highly negatively correlated, the pooled or averaged analysis will show quite a different picture than the two individual analyses. In fact, the pooled or averaged analysis would certainly not be representative of either subject.

While some may argue that such a problem can be minimized by dealing with larger sample sizes, some problems can still arise:

(a) "Outlier" or untypical responses will contribute to or affect the pooled or averaged analysis more than typical responses; and,

(b) One can still obtain a result that is not truly representative of the entire sample.

These two points have been demonstrated algebraically in Currim and Wittink (1979), and independently in DeSarbo, Carroll, and Lehmann (1981) where it was demonstrated that the coefficients obtained in the aggregate (pooled or averaged) analysis were the averages of the coefficients in the individual analyses.

Clearly, then, one could obtain very significant results in each of the individual regressions, but uncover no significant findings in the averaged or pooled analysis. This may indicate that the sample contained a very heterogeneous group of subjects with quite different preferences, or it may also indicate that the model specified did not fit the data from all subjects very well. Obtaining such results without fitting each individual's utility function (together with the averaged or pooled results) can therefore be quite inadequate.

There is also the problem of "outlier" or untypical responses affecting or contributing to the pooled or averaged analysis more than typical responses. Since the mean is extremely sensitive to extreme values, one subject's "outlier" responses could noticeably affect the coefficients, especially if the reason for the untypical data obtained was due to a recording or data input error.

One obvious alternative is to perform the conjoint analysis on the individual level (or by segment) and aggregate predictions via a choice simulator. In fact, most of the applied conjoint analyses performed in industry probably falls into this area of application. One certainly has the option to perform such analyses but for even moderate sample sizes, this task becomes quite tedious, especially when one attempts to *compare* the results over subjects. In addition, there are often inadequate degrees of freedom for meaningful estimation. Also, suppose one wishes to examine *why* subjects respond differently and collects various demographic, geographic, psychographic, and/or behavioristic (Kotler, 1980) information for each subject. How does one now attempt to explain response differences as a function for these new predictor variables? Again, one must perform more regressions and one has the problem of comparing the results over numerous subjects. In addition, such methods require additional analyses to uncover any relevant *interrelationships* among subjects and/or subject descriptor variables that might be quite useful for market segmentation and/or product positioning purposes.

## III. Three-Way Multivariate Conjoint Analysis

### A. *Objectives*

Our goal is to provide a model which will allow one to perform metric conjoint analysis when there are one or more dependent variables and/or one wishes to investigate individual differences and interrelationships in response. We wish to examine the interaction and contribution of effects due to subjects, experimental profiles, and the dependent variables in attempting to best fit the input responses. In addition, it is desirable to uncover potential interrelationships between and within these modes in order to better understand the structure, if any, in the data. To this end, we may also wish to constrain two of these three modes mentioned: profiles and subjects. For example, in order to examine why subjects respond differently, it might prove useful to constrain subject effects to be some function of various relevant psychographic and/or demographic descriptors. And, in the conjoint analysis frame of reference, it is useful to constrain profile effects to be some function of the various experimental attribute variables. In essence then, given a design matrix, observations on individual descriptors, and observations on the dependent variables for each subject, Three-Way Multivariate Conjoint Analysis will render insight into:

(1) Subject response differences and interrelationships;

(2) Subject response differences as a function of prescribed predictor variables;

(3) Experimental profile effects;

(4) Experimental profile effects as a function of the manipulated attributes à la conjoint analysis;

(5) Dependent variable effects and interrelationships;

(6) Dimensionality of the data.

### B. *Assumptions*

It is assumed that the response data collected is metrically scaled, i.e., it is measured on at least an interval scale. In addition, it is assumed that the form of the contraints on the subject and profile modes is linear. The model, as we shall demonstrate shortly, is multiplicative concerning subject, profile, and dependent variable effects and thus assumes a particular form of interaction effects amongst these various mode effects. It is additive within experimental profile and/or subject attributes. Its basic structure appears in Figure 1.

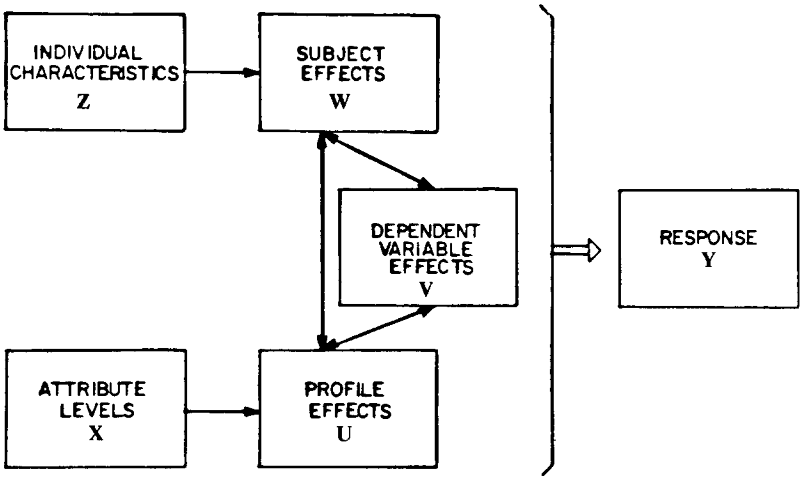


FIGURE 1. Three-Way Multivariant Conjoint Analysis Model.

## C. *The Models*

Let:

$Y_{ijk}$ = the response of the $i$th subject to the $j$th experimental full-profile on the $k$th dependent variable, $i = 1, \ldots, n, j = 1, \ldots, m, k = 1, \ldots, q$;

$Z_{is}$ = the value of subject $i$ on the $s$th descriptor variable, $s = 1, \ldots, p_1$;

$X_{jr}$ = the value of the $j$th experimental profile on the $r$th attribute, $r = 1, \ldots, p_2$;

$W_{it}$ = the weight of the $i$th person on the $t$th dimension or factor (this can be thought of as the importance of dimension $t$ to subject $i$), $t = 1, \ldots, T$;

$U_{jt}$ = the value of the $j$th stimulus profile on the $t$th dimension or factor;

$V_{kt}$ = the weight of the $k$th dependent variable on the $t$th dimension or factor (this can be thought of as the coefficient of the $t$th dimension for predicting the $k$th dependent variable);

$c_{st}$ = the weight of the $s$th descriptor variable on the $t$th dimension or factor;

$b_{rt}$ = the weight of the $r$th attribute on the $t$th dimension or factor;

$\alpha_i$ = a scale factor for subject $i$.

Then, the full model can be formulated as:

$$Y_{ijk} \approx \alpha_i \sum_t W_{it} U_{jt} V_{kt}, \tag{1}$$

where:

$$W_{it} = \sum_{s=1}^{p_1} Z_{is} c_{st}, \qquad U_{jt} = \sum_{r=1}^{p_2} X_{jr} b_{rt}. \tag{2}$$

Three-Way Multivariate Conjoint Analysis can be thought of as a straightforward generalization of the two-way factor or component analysis procedure. Recall that the two-way factor model for a given data point here can be written as follows:

$$Y_{jk} = U_{j1}V_{k1} + U_{j2}V_{k2} + \cdots + U_{jT}V_{kT} + \text{error}_{jk}. \quad (3)$$

Let us consider an interpretation of equation (3) in terms of a two-way matrix of (averaged) ratings on a number of dependent variables for a group of products (either actual or hypothetically defined via conjoint analysis). Each rating in the array would presumedly be an average based on a number of subjects. With this type of data, $Y_{jk}$ would represent the rating of product $j$ on variable $k$, $U_{j1}$ would represent the factor loading of the $j$th product on the first "latent" factor or dimension—how much the first factor/dimension of evaluation influences ratings of product $j$. The $V_{k1}$ coefficient would represent the loading of the $k$th variable on factor/dimension one—how much that dependent variable's ratings were influenced or reflected by factor/dimension one.

Three-Way Multivariate Conjoint Analysis has the form:

$$Y_{ijk} = W_{i1}U_{j1}V_{k1} + W_{i2}U_{j2}V_{k2} + \cdots + W_{iT}U_{jT}V_{kT} + \text{error}_{ijk}. \quad (4)$$

Here, $Y_{ijk}$ would represent the rating of product $j$ on dependent variable $k$ by subject $i$. (With this model, it is no longer necessary to average over subjects.) On the right hand side of the equation, $U_{jt}$ and $V_{kt}$ have the same interpretation as before, representing product loadings and dependent variable loadings. However, there is now an additional coefficient in each term, the $W_{it}$ coefficient. This represents the salience or importance of that particular evaluative factor or dimension for subject $i$. Thus, in this three-way model, the contribution of a given factor/dimension to a particular $Y_{ijk}$ rating is determined by three facets:

(a) the amount that the product reflects that factor/dimension (the $U_{jt}$ term);

(b) the amount that the given dependent variable on which the product is being rated reflects that factor/dimension (the $V_{kt}$ term); and,

(c) the amount that the individual performing the ratings is sensitive to that factor/dimension (the $W_{it}$ term).

Thus, as will be described shortly, we attempt to find the "best" values of $W_{it}$, $U_{jt}$, and $V_{kt}$ which, when combined multiplicatively, comes as "close" to the $Y_{ijk}$ (in a prespecified dimensionality $T$) as possible. In addition, we may choose to investigate possible sources of explaining why different subjects respond differently as well as why different products are evaluated differently. This is done here by constraining **W** to be a linear function of selected background variables such as demographics, psychographics, purchase history, etc., and by constraining **U** to be a linear function of product features as in conjoint analysis.

Thus, one can derive information concerning subject response differences ($\alpha_i W_{it}$) as a function of prescribed descriptor variables ($Z_{is}$), and examine what effects ($c_{st}$) these predictors have on individual differences. Similarly, one can obtain insight into experimental profile effects ($U_{jt}$) as a function of the designated attribute manipulated in the design ($X_{jr}$), and examine what effects ($b_{rt}$) these variables have on stimulus profile effects. Finally, one uncovers information as to the effect of the various dependent variables ($V_{kt}$). All of this is done in a prescribed dimensionality which is common across all modes and provides information as to the underlying structure of the data. In addition, the multiplicative model in (1) also accounts for possible interactions (see Harshman, 1980) among these three modes which may be of extreme importance in many conjoint analysis applications.

Four nested versions of this model are examined to investigate the impact of various additional parameters and constraints. Model I is the purely unconstrained version where only **W**, **U**, and **V** are estimated. Model II estimates these matrices with the linear constraints placed on **U**. Model III estimates these matrices with the linear constraints placed on *both* **U** *and* **W**. Finally, Model IV is basically Model III with the estimation of additional subject parameters $\boldsymbol{\alpha}$. This "hierarchy of models" will be discussed in detail in the next section.

## D. *The Four Models*

Four nested versions of the model (1) are examined to investigate the "tightness" of the constraints (2) and the contribution of the $\alpha_i$ parameters. Appendix I contains the mathematical formulations for each of the four models.

1. *Model* I: *Unconstrained.* The first model is the unconstrained CANDECOMP model:

$$Y_{ijk} \approx \sum_{t=1}^{T} W_{it} U_{jt} V_{kt}, \tag{5}$$

where subject descriptor variables and design independent variables are ignored as constraints for $W_{it}$ and $U_{jt}$. The algorithm utilized here is an *asymmetric* version (see DeSarbo and Carroll, 1980) of the standard alternating least-squares or NILES (Wold, 1966) procedure, also utilized in the CANDECOMP model (Carroll and Chang, 1970). Given the **Y**'s and a specified value of $T$, we find the **W**'s, **V**'s and **U**'s yielding a best least-squares fit to the **Y**'s. Given the three-way model presented in equation (1), we may, given current estimates of two sets of parameters (say the $U_{jt}$'s and $V_{kt}$'s), find an exact least squares estimate of the third set by linear regression methods.

The NILES (Nonlinear Iterative Least Squares) procedure for estimation in this case amounts to iterating this least squares estimation procedure, i.e., estimating the **W**'s (with **U**'s and **V**'s fixed) by least squares methods, then the **U**'s (with **W**'s and **V**'s fixed) and so on, around the iterative cycle until convergence in a goodness-of-fit measure occurs. Note, there is no proof that this process will converge, nor that, if it does, it will converge to the global (rather than a merely local) optimum solution. Nonetheless, it seems, empirically, "to work extremely well, and to be almost wholly free of local minima and similar problems" (Carroll and Chang, 1970). Monte Carlo work on this model performed by DeSarbo and Carroll (1980) also tends to support this claim. While estimation via nonlinear programming methods is also possible here, the amount of computation required for this would greatly exceed that of this NILES procedure. In addition, the local minimum problem would still exist.

This algorithm will render least-squares estimates of subject effects ($W_{it}$), experimental profile effects ($U_{jt}$), and dependent variable effects ($V_{kt}$) in a prescribed dimensionality. While the most general model (5) is perhaps the least interesting of all four models in our methodology since it does not provide part-worths or utilities to the design variables nor information as to the importance of subject descriptor variables, it does provide an upper bound for the four models in terms of goodness-of-fit since it is unconstrained.

2. *Model* II: *Constraints on Profile Effects.* The second model can be denoted as:

$$Y_{ijk} \approx \sum_{t=1}^{T} W_{it} U_{jt} V_{kt},$$

where:

$$U_{jt} = \sum_{r=1}^{p_2} X_{jr} b_{rt}. \tag{6}$$

In essence, Model II differs from Model I by constraining the experimental profile effects ($U_{jt}$) to be linear functions of the experimental design independent variables ($X_{jr}$), in the spirit of conjoint analysis. This can be viewed as an asymmetric three-way modification of the CANDELINC model (Carroll, Pruzansky, and Kruskal, 1979), where only one of the modes ($U_{jt}$) is constrained.

Here, one obtains estimates of subject response differences (**W**), dependent variable effects and interrelationships (**V**), experimental profile effects and interrelations (**U**), and the part-worths or utilities (**B**) for the independent design variables.

3. *Model* III: *Constraints on Both Profiles and Subject Effects.* Model III can be written as:

$$Y_{ijk} \approx \sum_{t=1}^{T} W_{it} U_{jt} V_{kt},$$

where:

$$U_{jk} = \sum_{r=1}^{p_2} X_{jr} b_{rt}, \qquad W_{it} = \sum_{s=1}^{p_1} Z_{is} c_{st}. \tag{7}$$

Here, constraints have been placed on both subject and profile modes. The objective in doing this is to obtain both the conjoint part-worths via constraints on **U** and response style differences as a function of prespecified descriptor variables. This latter addition can be very useful for market segmentation and/or product positioning purposes where individual differences vis-à-vis specified background variables can be utilized for the creation of target market segments and associated product positioning strategies within such segments. Although the literature (e.g., Moore, 1980) warns *against* the use of demographics alone to predict behavior in many types of conjoint analysis studies, demographics when combined with psychographics, product usage, purchase occasion, benefits sought, user status, etc., can provide (Kotler, 1980) an effective set of variables upon which to formulate initial segmentation and positioning strategies. DeSarbo and Rao (1982) illustrate the use of such information in an optimal positioning model for new telecommunications equipment.

Thus, Model III provides us with:

(a) the effects and interrelationships between the dependent variables (**V**);

(b) the effects and interrelationships between subjects (**W**);

(c) the importance of various descriptor variables concerning individual differences (**C**);

(d) the effects and interrelationships between experimental profiles (**U**);

(e) the part-worths of the independent design variables and their levels (**B**).

4. *Model* IV: *Model* III *with Additional Subject Parameters.* The mathematical formulation for Model IV can be written as:

$$Y_{ijk} \approx \alpha_i \sum_{t=1}^{T} W_{it} U_{jt} V_{kt},$$

where:

$$W_{it} = \sum_{s=1}^{P_1} Z_{is} c_{st}, \qquad U_{jt} = \sum_{r=1}^{P_2} X_{jr} b_{rt}, \quad \alpha_i \geqslant 0. \tag{8}$$

Model IV is essentially a generalization of Model III with additional parameters $\alpha_i$, $i = 1, \ldots, n$, which have the effect of stretching or shrinking the derived subject vectors. Carroll, Pruzansky, and Kruskal (1979) observed a profound degeneration of derived configuration and fit measures when placing constraints on subjects. This was also found in analyses by Carroll, Green, and Carmone (1976) and Green, Carroll, and Carmone (1976) with a two-way application of CANDELINC. According to Carroll, Pruzansky, and Kruskal (1979), this empirically observed degeneration of configuration and fit measures can be related to a theoretical argument against putting constraints of this type on entities which are interpreted as vectors, rather than as points. Since the directional information is of primary interest in the case of such vector-like entities, it would seem that only constraints should be considered that are invariant under changes in length of individual vectors. In particular, it should be clear that the linear constraints imposed in CANDELINC are *not* invariant under such length altering transformations. An easy way to see this is to imagine subject points arrayed in a regular square or rectangular lattice arrangement in the positive quadrant of a two-dimensional space. If we now change the lengths, say by normalizing all vectors to unit length, this lattice structure will be almost wholly destroyed. This limitation is also applicable to the three-way nonsymmetric CANDELINC model adapted in Model III where linear constraints are also placed on the subject mode. To this end, Model IV has been developed where an additional stage of computation is programmed to estimate these $\alpha_i$ stretching/shrinking parameters in order to restore subject vector lengths in the configuration and enhance associated goodness-of-fit measures.

Model IV thus renders the estimates **W**, **V**, **U**, **C**, and **B**, as well as $\alpha_i$, $i = 1, \dots, n$, subject parameters.

5. *Nested Feature of the Four Models.* The degrees of freedom for the four different models are:

$$\begin{aligned} &\text{Model I:} && (n + m + q - 2)T \\ &\text{Model II:} && (n + p_2 + q - 2)T \\ &\text{Model III:} && (p_1 + p_2 + q - 2)T \\ &\text{Model IV:} && (p_1 + p_2 + q - 2)T + (n - 1). \end{aligned}$$

The respective degrees of freedom for the residual would merely be these quantities subtracted from the total number of data points ($nmq$). Model I clearly has the most model degrees of freedom since the highest number of parameters must be estimated. Assuming $m > p_2$, Model II is nested within Model I since constraints are placed on **U**.

Assuming $T > (n - 1)/(n - q)$, Model IV becomes nested in Model II. If this condition does not hold, then Model IV is underdetermined and is essentially equivalent to Model II.[1] Finally, Model III is nested in Model IV.

[1] For true nesting of the four models, a necessary condition is that:

$$df_I > df_{II} > df_{IV} > df_{III},$$

where $df$ refers to the model degrees of freedom. Thus for $df_{II} > df_{IV}$, the condition $T > (n - 1)/(n - p_1)$ must hold or else Model IV is underdetermined and equivalent to Model II. This condition is derived from the definitions of $df_{II}$ and $df_{IV}$ in Table 2 and from the nature of the model parameters involved.

Unfortunately, because of the lack of adequate statistical distribution theory, one can not "legitimately" utilize standard model comparison tests to investigate which model of the four best fits the data. However, as in PREFMAP, "pseudo-$F$" statistics are given for all pairs of models as rough indications of which model might be most appropriate. Here, one can formulate (Green, 1978):

$$\hat{F} = \frac{R_f^2 - R_r^2}{1 - R_f^2} \cdot \frac{df_f}{df_r - df_f}, \tag{9}$$

where:
$R_f^2$ = goodness of fit for full model;
$R_r^2$ = goodness of fit for restricted model;
$df_f$ = model degrees of freedom for full model;
$df_r$ = model degrees of freedom for restricted model;
$\hat{F}$ = pseudo-$F$ statistic,
and compare $\hat{F}$ vs an $F$ with $(df_f - df_r)$ degrees of freedom for numerator and $df_f$ degrees of freedom for denominator. Again, because of the lack of adequate distribution theory here, this is only meant to be a *rough* indication for selecting one of the four models. Clearly, one must also examine interpretation, split-half analyses, etc. in selecting one of the four models.

## IV. Application

### A. *Study Description*

The study employed was an attempt to describe patterns in the criteria used by purchasing managers in selecting suppliers for different types of products. Specifically four types of choice criteria (four dependent variables) were considered:

1. Economic (relative cost outlays associated with the purchase),
2. Technical/performance,
3. Integrative (cooperation from suppliers),
4. Certainty of supply (supplier capability in responding to changes in the environment).

In order to see if the relative importance of these varied across situations, products were classified into four basic categories (from which the experimental design is to be created as will be shown shortly):

1. Routineness of product (standard—nonstandard),
2. Complexity (simple—complex),
3. Novelty of application (standard—novel),
4. Dollar commitment involved (low—high).

The focus of the study was to see if the product types impacted on the relative importances of the four choice criteria.

Data were gathered by means of a mail survey to members of the National Association of Purchasing Managers. The questionnaire required managers to

TABLE 1
*Purchase Style Variables*

Please indicate your degree of agreement with the following statements by circling a "1" if you strongly disagree, a "6" if you strongly agree, or somewhere in between depending on how much you agree with the statement.

| | Strongly Disagree | | | | | Strongly Agree |
|---|---|---|---|---|---|---|
| Compared to other purchasing managers, I am more compulsive about carefully considering alternatives before I buy. | 1 | 2 | 3 | 4 | 5 | 6 |
| Compared to other purchasing managers, I am more concerned about how others will react to what I recommend or purchase. | 1 | 2 | 3 | 4 | 5 | 6 |
| Compared to other purchasing managers, I tend to spend less time in reaching a purchase decision. | 1 | 2 | 3 | 4 | 5 | 6 |
| Compared to other purchasing managers, I tend to focus more heavily on objective criteria. | 1 | 2 | 3 | 4 | 5 | 6 |
| Compared to other purchasing managers, I tend to be more loyal to current suppliers. | 1 | 2 | 3 | 4 | 5 | 6 |
| Compared to other purchasing managers, I tend to be technically better informed about products. | 1 | 2 | 3 | 4 | 5 | 6 |
| Compared to other purchasing managers, I am more likely to avoid taking risks when buying. | 1 | 2 | 3 | 4 | 5 | 6 |

allocate ten points among the four choice criteria for each of eight hypothetical products. Definitions of both the choice criteria and product dimensions were provided to the respondents. The eight hypothetical products formed an orthogonal array (for main effects estimation) from the 16 possible combinations of the four product categories/attributes. Respondents also rated themselves in terms of seven 6-point agree-disagree scales, listed in Table 1, which attempted to measure their purchasing style relative to other purchasing managers in terms of attributes such as loyalty to suppliers and thoroughness. They also provided data on such variables as years of experience, education, etc.

The survey was sent to a random sample of 600 purchasing managers. Within three weeks, 240 (40%) responded and 220 (35%) were complete (i.e., no missing data) and are used here. More extensive discussion of both the data and the results appear in Lehmann and O'Shaughnessy (1980). The objective of the study was to explain patterns in the criteria used in selecting suppliers where different products and different purchasing managers are involved.

B. *Analysis of a Selected Subsample*

In order to facilitate the illustration of a Three-Way Multivariate Conjoint Analysis and because of local computer usage constraints,[2] we examined a "representative" subset of the $N = 220$ subjects. The 220 subjects were cluster analyzed via Johnson's compact hierarchical method (Johnson, 1967) on Euclidean distances (proximities) in two separate analyses. In analysis one, the proximities for the cluster analysis were generated via a subjects by dependent variables input matrix, and a resulting dendrogram (hierarchical clustering tree) was obtained clustering the $N = 220$ subjects. In the second analysis, the proximities for the hierarchical clustering were obtained from a subject by descriptor variable (purchasing style) input matrix, and a second dendrogram of the $N = 220$ subjects was obtained. From a cursory visual inspection of each dendrogram, it appeared that fifteen or sixteen clusters could be found in each analysis. Each tree was "cut" at sixteen clusters (Becker and Chambers, 1980), and the $N = 220$ subjects were then classified, first in the analysis with the dependent variables, and then in the other with the descriptor variables. A matching procedure was then initiated in order to pick sixteen subjects of the total 220 that were members of both sets of sixteen clusters. While it is also likely that other groups of sixteen subjects could also pair different clusters so that a different subsample of sixteen subjects could be drawn, only one set of sixteen subjects was selected on this basis in order to merely *illustrate* the use of the technique. As with all subsampling plans, it is conceivable that different sets of sixteen subjects could render different results. Regardless, substantive conclusions based on this small a sample are clearly tenuous.

1. *Univariate Conjoint Analysis.* A pooled dummy variable regression analysis was performed separately for each dependent variable. The analysis was performed for both the total sample (see DeSarbo, Carroll, and Lehmann (1981) for results) and the subsample of 16 subjects. Here (as in the conjoint analysis for the entire $N = 220$ subjects) we found that economic criteria become more important as a product becomes standard, simple, and with a typical application (Table 2). The direct opposite held for performance criteria, which were viewed as being more important for nonstandard, complex products with novel applications. Neither cooperation nor certainty of supply importance variables were related to any of the four product variables in this set of analyses.

2. *Individual Differences.* Pooled regression analyses were also run for each dependent variable as a function of the seven purchasing style variables. Again, the analysis was performed for both the total sample (see DeSarbo,

[2]The Three-Way Multivariate Conjoint Analysis procedure was initially programmed in the APL computer language, which because of its structure and operating environment (interpretive execution) is not suited to handling large arrays. A Fortran version of the program will soon be devised.

TABLE 2
*Univariate Conjoint Analysis on n = 16 Subjects*

| | Regression Coefficients | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Dependent Variable | $b_0$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | S.E. | $R^2$ | Adjusted $R^2$ | $F$ |
| Economic | 5.39 | − 1.13** | − 1.38** | −.97** | −.09 | 2.20 | .18 | .15 | 6.76** |
| Performance | 1.52 | 1.34** | 1.69 | .75** | −.22 | 1.82 | .29 | .27 | 12.75** |
| Cooperation | 1.49 | −.14 | −.02 | .27 | −.05 | 1.16 | .02 | .00 | 0.553 |
| Certainty | 1.60 | −.08 | −.39 | −.05 | .36* | 1.22 | .04 | .00 | 1.21 |

* indicates significance at $p < .05$
** indicates significance at $p < .01$
$b_0$ not tested for significance

$b_0$ constant
$b_1$ Economic ($X_1$)
$b_2$ Performance ($X_2$)
$b_3$ Integrative ($X_3$)
$b_4$ Adaptive ($X_4$)

Carroll, and Lehmann (1981) for results) and the subset of 16 subjects (in Table 3). Subjects for whom economic criteria are important tend to be: more concerned about how others react, less loyal to current suppliers, and less informed about products. When performance criteria are important, subjects tend to be: less concerned how others react, more loyal to current suppliers, and better informed about products. When cooperation is important, managers tend to: spend more time in reaching a purchase decision, be better informed about products, and be less likely to avoid taking risks. The fourth dependent variable, certainty of supply, is unrelated to any of the purchasing style independent variables.

Unlike the univariate conjoint analysis above, the results here do not closely resemble the corresponding analysis performed for the entire sample. This may primarily be due to the relatively poor relationships between the four dependent variables and the seven independent variables in the data. Also, because only one subject was selected per cluster, the different results obtained could be due to sampling error.

### C. *Analysis via Three-Way Multivariate Conjoint Analysis*

1. *Major Options Selected.* The analysis was run in 1, 2 and 3 dimensions with the convergence tolerance set at .00001 and 100 maximum number of iterations for the NILES procedure. In all cases, the four models were requested. The raw data, **Y** (16 subjects × 8 profiles × 4 dependent variables), were row centered to remove the effects of the row means which, since they are constrained to be the same for all rows for this task, really do not convey any useful information. Note that in this special case, the row means are all equal to 2.5, which is also the overall grand mean of the three-way array **Y** (as

TABLE 3

*Role of Individual Descriptors for $n = 16$ Subjects*

| Dependent Variable | Regression Coefficients $b_0$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ | S.E. | $R^2$ | Adjusted $R^2$ | $F$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Economic | 5.20 | .18 | .56** | .21 | −.09 | −.39** | −.80** | .17 | 2.18 | .22 | .17 | 4.85** |
| Performance | 2.16 | −.10 | −.36** | −.13 | −.02 | .40** | .36* | .04 | 2.05 | .12 | .07 | 2.44* |
| Cooperation | .80 | −.04 | −.11 | −.17** | .11 | .01 | .46** | −.22** | 1.05 | .21 | .16 | 4.56** |
| Certainty | 1.83 | −.04 | −.09 | .09 | −.01 | −.03 | −.02 | .02 | .121 | .03 | .00 | 0.55 |

*indicates significance at $p < .05$
**indicates significance at $p < .01$
$b_0$ not tested for significance

$b_0$ constant
$b_1$ Purchase Style Variable $Z_1$
$b_2$ Purchase Style Variable $Z_1$
$b_3$ Purchase Style Variable $Z_3$
$b_4$ Purchase Style Variable $Z_4$
$b_5$ Purchase Style Variable $Z_5$
$b_6$ Purchase Style Variable $Z_6$
$b_7$ Purchase Style Variable $Z_7$

TABLE 4

*Goodness of Fit Measures For the Two Dimensional Solutions*

| | V-Unrestricted | V-Restricted |
|---|---|---|
| Model I: | .669 | .528 |
| Model II: | .662 | .511 |
| Model III: | .522 | .428 |
| Model IV: | .582 | .489 |

well as each two-dimensional profile × dependent variable mean). Intercept terms were estimated for each of the constraint conditions. Analyses were also performed with prespecified restrictions on the form of **V** to aid in the interpretation of the results. Based upon goodness-of-fit, interpretability, and correlations between dimensions, the two-dimensional solution (see Harshman, 1980) appeared clearly to be the appropriate representation and will be reported here.

2. *Results.*

a. *Unrestricted* **V**. Table 4 presents the variance accounted-for statistics for Models I, II, III, and IV in two dimensions. While the drop in goodness-of-fit is small (.007) in proceeding from Model I to Model II, indicating that the profile constraints are not all that "tight", the drop in variance accounted-for in going from Model II to III is quite large (.140). This indicates that the subject constraints are "binding" and affect the resulting solution. This also provides some evidence for the solution deterioration and possible configuration distortion phenomena associated with Model III's constraints on subjects mentioned earlier. Note the 6% increase in the variance accounted-for statistic in going from Model III to Model IV, indicating that the stretching/shrinking estimation phase does tend to alleviate the problem of placing constraints on subjects. Since we are interested in examining *both* profile and subject constraints, we shall focus on the examination of the results of Model IV,[3] although, as we shall see, because of the lack of relationship between purchase style variables and individual differences, Model II according to (7) may be a more parsimonious description of the structure in the data.

(1) Dependent Variable Effects. Figure 2 presents a plot of the derived configuration for dependent variable effects: **V**. The first dimension is clearly dominated by economic(−) vs. performance(+) criteria. The second dimension is dominated by economic criteria, but with a somewhat moderately sized loading on cooperation.

(2) Betas for Design Matrix. Figure 3 presents a plot of the derived solution for the betas (constraints) on the design matrix: **B**. As in the traditional analysis results, performance criteria (economic criteria) become more (less) important as one has a nonstandard, complex, novel product.

(3) Betas for Subject Descriptors. Figure 4 shows a plot of the betas on the individual descriptors: **C**. Here, when performance criteria are important,

[3] Model IV solutions converged in two iterations indicating that the resulting **B**, **C**, **U**, **W**, and **V** matrices are identical to Model III's.

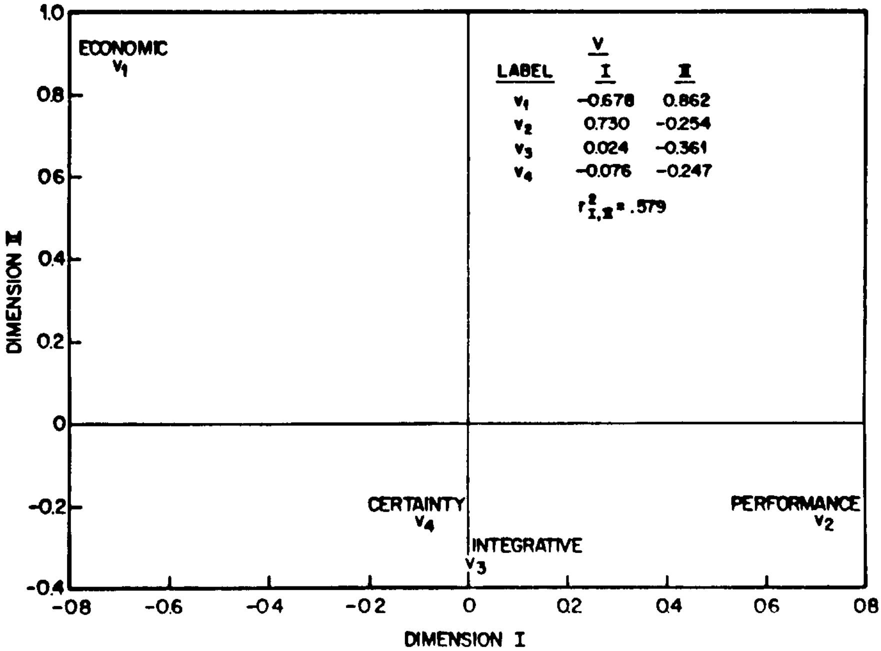


FIGURE 2. Dependent Variable Effects V.

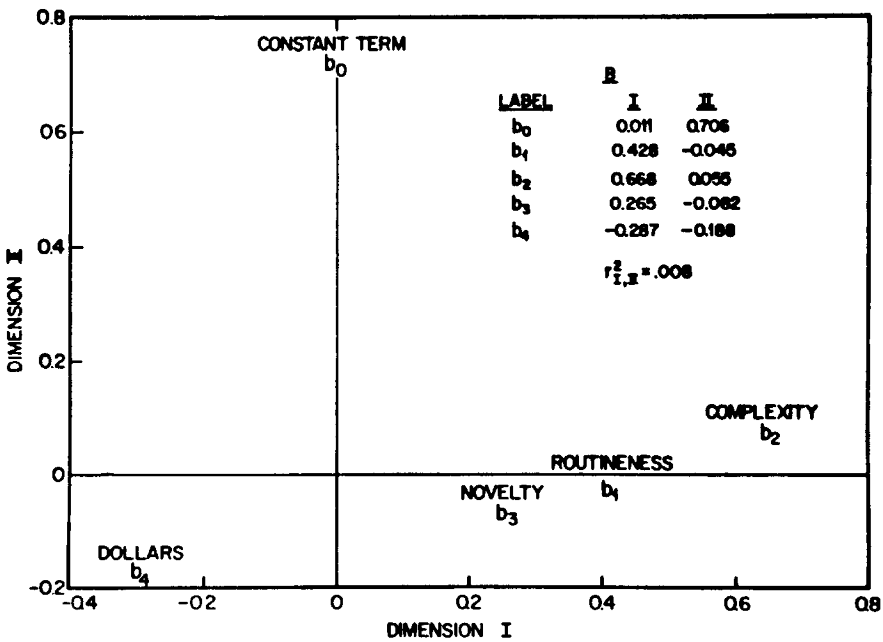


FIGURE 3. Betas (B) on Design Matrix X.

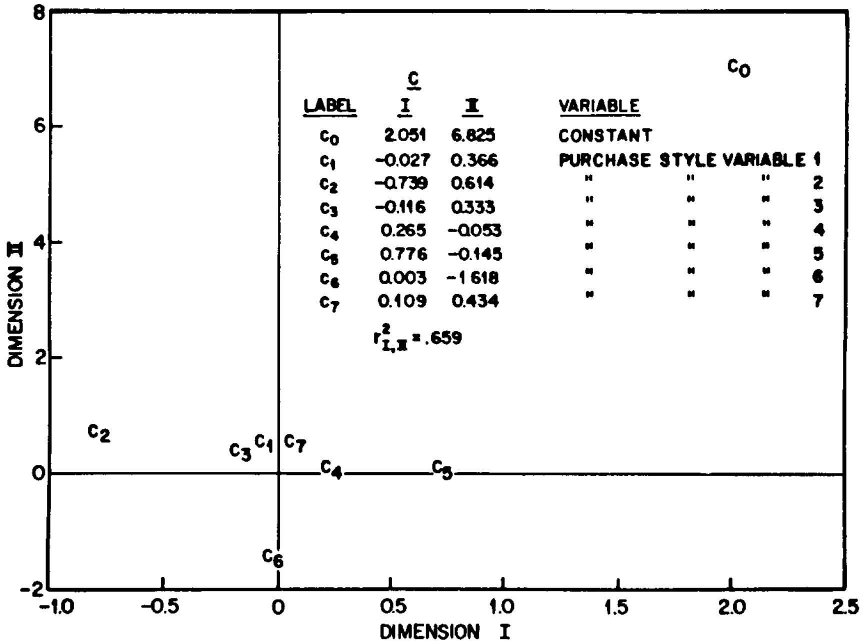


FIGURE 4. Betas (C) on Individual Descriptors **Z** Matrix.

managers tend to be: less concerned how others will react and more loyal to current suppliers. The opposite holds true for the case when economic criteria are important. Again, the results seem to substantiate the findings obtained in the traditional analysis.

(4) Subject Response Differences. Figure 5 presents a plot of the individual effects solution: **W**. Here, one can see that subjects 3, 6, and 16 tend to load heavily on the first dimension, while subjects 4, 5, 7, 8, 9, 11, 13, and 16 load highly on the second dimension. This gives us insight as to how subjects weighted these criteria differently—an aspect not addressed in traditional analyses.

(5) Profile Effects. Figure 6 presents a plot of the derived solution for the design profile effects: **U**. Profiles 4, 6 and 8 have greatest impact on the first dimension, while profile 1, 4, and 6 have greatest impact on the second dimension. This is another aspect not addressed in traditional analyses.

(6) Subject Adjustment Factors. Figure 7 presents the vector adjustment factors $\alpha_i$, $i = 1, \ldots, 16$, for the sixteen subjects. The rather substantial deviation from 1 that these numbers exhibit suggests that Model III may in fact substantially distort subject vectors in this example as suggested earlier. This is especially shown with respect to subject 14's high $\alpha$ value which, because of its size, had to be omitted from the plot in Figure 7.

b. *Restrictions on* **V**. Since the derived dimensions for the various effects involve linear combinations of the original variable comprising these modes, an option has been built in these models to allow for the user to restrict the

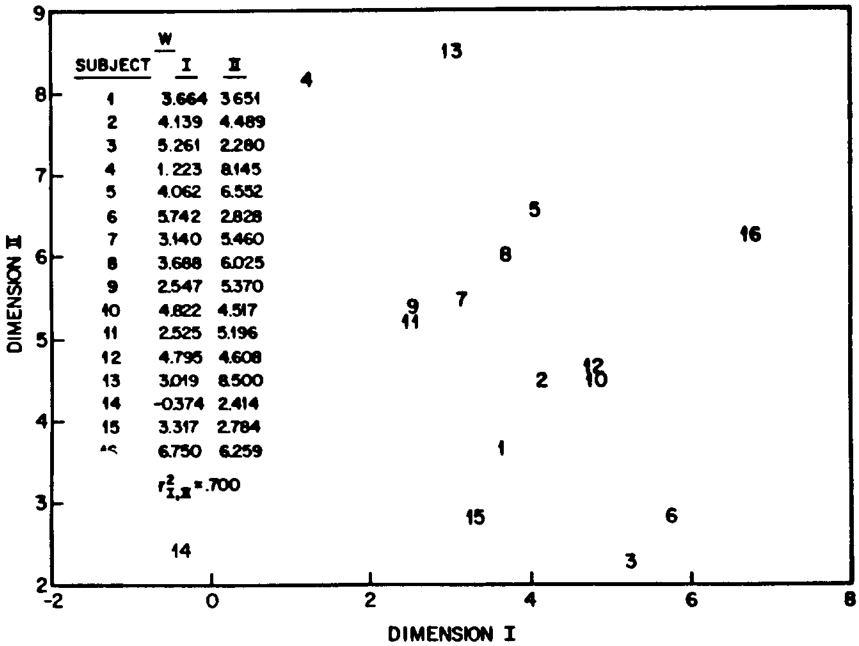


FIGURE 5. Subject Effects W.

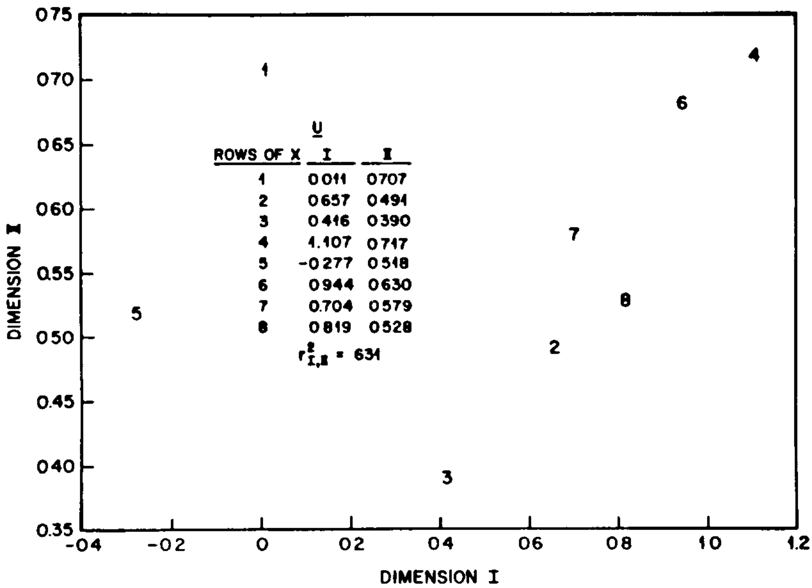


FIGURE 6. Design Profile Effects U.

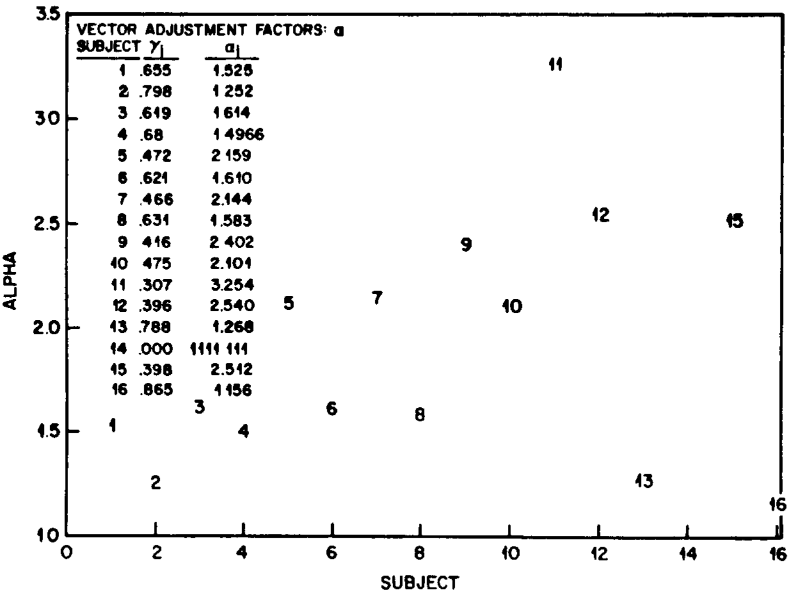


FIGURE 7. Alpha Cumulative Effects.

dependent variable mode (**V**) to remain fixed throughout the analysis. Because of the apparent dominance of the economic and performance variables in the Three-Way Multivariate Conjoint Analysis as noted above, another two-dimensional solution was run with the following restrictions on **V** (the $4 \times 2$ matrix of dependent variable (choice criteria) effects):

$$\mathbf{V} = \begin{bmatrix} a & 0 \\ 0 & b \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

where $a$ and $b$ are any real constants. (Actually, $a$ and $b$ are both set equal to 1 in the algorithm, and it is the normalization procedure at the end of this estimation phase which can make $a$ and $b$ any real constants.) In essence, this restriction forces the first dimension of the solution to focus upon the economic criteria variable above, while the second dimension examines the performance criteria variable alone. This has the effect of orienting the **B**, **C**, **W** and **U**, matrices to solely these two dependent variables. The results (presented in DeSarbo, Carroll, and Lehmann, 1981) are essentially the same as in the unrestricted case.

c. *Discussion.* We have learned that two dimensions—one related to Economic criteria and one related to Performance criteria—adequately describe the structure of the data. This was also indirectly demonstrated in the traditional conjoint analyses performed. As in the traditional analyses, purchase style descriptor variables are not strong predictors of individual differences—a result supported by much Marketing literature (e.g., Moore (1980)). However, the Three-Way Multivariate Conjoint Analysis gives us more information. It examines which subjects weight the two dimensions similarly and which do not (Figure 5: **W**), hence aiding in benefit segmentation. We also can examine the interdependence relations which exist between our dependent measures via Figure 2 (**V**). Finally, we are able to examine effects and interrelationships among the various experimental profiles (Figure 6: **U**) to aid in interpretation.

## V. Conclusion

### A. *Other Marketing Applications*

An extension to traditional metric conjoint analysis has been developed with an illustration to demonstrate the method. Three-Way Multivariate Conjoint Analysis is not only restricted to applications examining industrial buying behavior. This new technique might be employed in advertising, pricing, product policy, etc. For example, Three-Way Conjoint Analysis could be employed in determining appropriate advertising copy. Consider a design matrix being used where the independent variables relate to descriptors of print advertising, e.g., color vs. black and white, size, appeal, etc. The dependent variables could be elements of the hierarchy of effects such as awareness, knowledge, preference, conviction, intention to buy. The third mode could relate to individual differences as a function of various measured psychographic/demographic descriptor variables as well as usage patterns. This would make possible the investigation of which types of advertising copy might be most appropriate for certain types of market segments.

One could also employ Three-Way Multivariate Conjoint Analysis with respect to new product design. Here, one could employ a design matrix to manipulate levels of features or attributes of a newly proposed product. The dependent variables of interest could be overall preference, intention to buy, and ratings on a number of specific product benefits. Again, a number of subjects representing different market segments could be utilized, given background psychographic and/or demographic descriptors.

### B. *Methodological Extensions*

A useful extension of this newly developed method would be to adapt it to handle the nonmetric case where the data are merely ordinally scaled. This could be done by replacing[4] the regression phase described for estimating the $\alpha_i$'s in Model IV by a monotone regression phase in which Kruskal's (1964a, b) MFIT routine, for example, is used to find the best fitting monotone function of each subject's data—that is, the monotone function agreeing best with the

[4]Note that Model IV becomes irrelevant as soon as one considers the possibility of such a nonmetric analysis since a general monotonic function (say, separately for each subject) is sufficiently general to absorb the $\alpha_i$ parameters.

predicted values from the (current) model estimates in a least-squares sense. Just as in fitting Model IV by alternating between fitting Model III and fitting the $\alpha_i$ parameters via simple homogeneous linear regression (i.e., linear regression without a constant term), we could fit any one of the other three models (I, II, or III) nonmetrically by alternating between fitting that model and using monotone regression separately for each subject.

This, of course, assumes that the data are "matrix conditional"—that is, the data are comparable *within* each subject's stimulus profile × dependent variable matrix, but not comparable *between* subjects. Other assumptions about the data—e.g., that the data are row conditional within each subject's data matrix (i.e., that a single subject's judgments are not comparable across different dependent variables)—would, of course, lead to different estimation schemes. It is unlclear how serious local optimum problems would be in this case.

## C. *Limitations*

There are some limitations of the Three-Way Multivariate Conjoint Analysis Model. Clearly, there is the potential for interpretation problems. Whenever one deal with linear composites in a canonical correlation or multidimensional scaling framework, interpretation is sometimes risky. However, Three-Way Multivariate Conjoint analysis does enable one to build in a priori information (restictions on V) to enhance interpretation.

There is also the potential problem of locally optimum solutions since the alternating least-squares algorithm employed in all four models can only guarantee a locally optimum solution. One method to investigate this possibility is to run an analysis two or three times, each using a different starting solution.

Finally, there are also limitations which exist with respect to the structure of the models themselves. For example, subject $x$ dependent variable two-way interactions are not measurable in the models per se. Harshman (1980) discusses various types of preprocessing and their effects of "smoothing" out such unmeasurable effects.

In spite of these potential drawbacks, the approach seems both sufficiently intriguing and potentially efficient to warrant further investigation.

# Appendix I

## A. *Model* I: *Unconstrained*

Given the three-way model presented in equation (1), we may, given current estimates of two sets of parameters (say the $U_{jt}$'s and $V_{kt}$'s), find an exact least squares estimate of the third set by linear regression methods.

This can be seen reformulating the problem as:

$$Y_{is}^{**} = \sum_t W_{it} G_{st}, \tag{A-1}$$

where:

$$Y^{**}_{is} = Y_{i(jk)},$$
$$G_{st} = U_{jt}V_{kt}, \tag{A-2}$$

and $s$ is a subscript that ranges over all $mq$ values of $j$ and $k$.

In matrix notation, this can be formulated as:

$$\mathbf{Y^{**}} = \mathbf{WG'}, \tag{A-3}$$

and we can estimate $\mathbf{W}$ by:

$$\hat{\mathbf{W}} = \mathbf{Y^{**}G(G'G)}^{-1} \tag{A-4}$$

(amounting to postmultiplying by the right pseudoinverse of $\mathbf{G'}$).

B. *Model* II: *Constraints on Profile Effects*

Redefining $\mathbf{X} = [\mathbf{J}, \mathbf{X}]$, where $\mathbf{J} = [1, 1, \ldots, 1]'$, to account for optional intercept terms, we, as in CANDELINC, initially perform a singular value decomposition on $\mathbf{X}$, assuming of course that $m \geqslant p_2$. We obtain:

$$\mathbf{X} = \mathbf{P}_1\mathbf{\Delta}_1\mathbf{Q}'_1, \tag{A-5}$$

where:

$\mathbf{\Delta}_1$ = diagonal matrix of the square root of the eigenvalues of $\mathbf{X'X}$,

$\mathbf{Q}_1$ = matrix of eigenvectors $\mathbf{X'X}$,

$\mathbf{P}_1 = \mathbf{XQ}_1\mathbf{\Delta}_1^{-1}$.

Defining

$$\mathbf{X^*} = \mathbf{P}_1\mathbf{Q}'_1, \tag{A-6}$$

where $\mathbf{X^*}$ is the best (least-squares) orthonormal approximation[5] to $\mathbf{X}$, we

[5] Carroll, Pruzansky, and Kruskal (1979) show the necessity of this orthogonalization procedure in their article.

create **Y*** via:

$$Y^*_{abc} = \sum_i \sum_j \sum_k Y_{ijk} I^*_{ia} X^*_{jb} I^{**}_{kc}$$

$$= \sum_j Y_{ajc} X^*_{jb}, \tag{A-7}$$

where:

$I^*$ = an $n \times n$ identity matrix,

$I^{**}$ = a $q \times q$ identity matrix,

$Y^*$ = an $n \times (p_2 + 1) \times q$ array.

We then fit the CANDECOMP model:

$$Y^*_{abc} \approx \sum_{r=1}^{T} W^*_{ar} B^*_{br} V^*_{cr} \tag{A-8}$$

to **Y*** by utilizing the NILES procedure discussed in the previous section for obtaining the estimates **W***, **B***, and **V***. Now, $\mathbf{W}^* \equiv \mathbf{W}$ gives information as to subject differences. $\mathbf{V}^* \equiv \mathbf{V}$ renders insight into dependent variable effect differences. $\mathbf{U} = \mathbf{X}^*\mathbf{B}^* = \mathbf{XB}$, where:

$$\mathbf{B} = \mathbf{Q}_1 \mathbf{\Delta}^{-1} \mathbf{Q}_1' \mathbf{B}^*, \tag{A-9}$$

gives us information as to experimental profile effects and interrelationships (the rows of **X**) in terms of the original set of variables, while **B** provide the part-worths or utilities for the independent design variables (columns of **X**).

C. *Model* III: *Constraints on Both Profiles and Subject Effects*

Redefining both **X** and **Z**,

$$\mathbf{X} = [\mathbf{J}, \mathbf{X}]$$
$$\mathbf{Z} = [\mathbf{J}, \mathbf{Z}], \tag{A-10}$$

where again $\mathbf{J} = [1, 1, \ldots, 1]'$ to account for optional intercept terms, we perform a singular value decomposition on both **X** and **Z**, obtaining:

$$\mathbf{X} = \mathbf{P}_1 \boldsymbol{\Delta}_1 \mathbf{Q}_1$$

and

$$\mathbf{Z} = \mathbf{P}_2 \boldsymbol{\Delta}_2 \mathbf{Q}_2 , \tag{A-11}$$

as was done in Model II. Defining

$$\mathbf{X}^* = \mathbf{P}_1 \mathbf{Q}_1'$$

and

$$\mathbf{Z}^* = \mathbf{P}_2 \mathbf{Q}_2' , \tag{A-12}$$

we create **Y**** via:

$$Y_{abc}^{**} = \sum_i \sum_j \sum_k Y_{ijk} Z_{ia}^* X_{jb}^* I_{kc}^*$$

$$= \sum_i \sum_j Y_{ijc} Z_{ia}^* X_{jb}^* , \tag{A-13}$$

where **Y**** is *a* $(p_1 + 1) \times (p_2 + 1) \times q$ array. One now obtains the CANDECOMP model:

$$Y_{abc}^{**} \approx \sum_{r=1}^{T} C_{ar}^{**} B_{br}^{**} V_{cr}^{**} , \tag{A-14}$$

and utilizes the NILES procedure as in Models I and II to obtain **C****, **B**** and **V****. One can now solve for these coefficients in terms of the original variables via:

$$\mathbf{C} = \mathbf{Q}_2 \boldsymbol{\Delta}_2^{-1} \mathbf{Q}_2' \mathbf{C}^{**}$$

and

$$\mathbf{B} = \mathbf{Q}_1 \boldsymbol{\Delta}^{-1} \mathbf{Q}_1' \mathbf{B}^{**}, \tag{A-15}$$

Similarly, having the dependent variables effect $\mathbf{V} = \mathbf{V}^{**}$, one can also solve for both profile and subject effects via:

$$\begin{aligned} \mathbf{U} &= \mathbf{XB} = \mathbf{X}^*\mathbf{B}^{**} \\ \mathbf{W} &= \mathbf{ZC} = \mathbf{Z}^*\mathbf{C}^{**} \end{aligned} \tag{A-16}$$

D. *Model* IV: *Model* III *with Additional Subject Parameters*

Essentially, we initially perform a Model III analysis as discussed above. At the end of this first stage, we calculate:

$$\hat{Y}_{ijk}^{(1)} = \sum_{t=1}^{T} W_{it}^{(1)} U_{jt}^{(1)} V_{kt}^{(1)}, \tag{A-17}$$

where:

$\hat{Y}_{ijk}^{(1)}$ = the predicted value of $Y_{ijk}$ given estimates of **W**, **U**, and **V** on the first iteration;

$W_{it}^{(1)}$ = the estimated value of subject effects for subject $i$ on dimension $t$ on the first iteration;

$U_{jt}^{(1)}$ = the estimated value of profile effects of profile $j$ on dimension $t$ on the first iteration;

$V_{kt}^{(1)}$ = the estimated value of dependent variable effects for dependent variable $k$ on dimension $t$ on the first iteration.

The next stage involves performing $n$ simple regressions to solve for $\gamma_i$ via:

$$\hat{Y}_{ijk}^{(1)} \approx \gamma_i Y_{ijk}^{(1)}, \qquad i = 1, \ldots, n \tag{A-18}$$

where the dependent variable is the predicted values and the independent variable is the actual data. The $\gamma_i$, $i = 1, \ldots, n$ are constrained to be greater than or equal to zero (for reasons we shall soon explain). After this step, we redefine $Y_{ijk}$ via:

$$Y_{ijk}^{(2)} = \gamma_i Y_{ijk}^{(1)}, \tag{A-19}$$

and return to the Model III stage to derive estimates of **W**, **U**, **V**, **B** and **C**. At the end of this, we calculate:

$$\hat{Y}_{ijk}^{(2)} = \sum_{t=1}^{T} W_{it}^{(2)} U_{jt}^{(2)} V_{kt}^{(2)}, \tag{A-20}$$

and repeat this same process iteratively until we reach convergence in the appropriate goodness-of-fit measure.[6] This additional stage for estimating $\gamma_i(\alpha_i = 1/\gamma_i)$ has the effect of stretching/shrinking the data,[7] instead of the subject vectors, which basically accomplishes the same objective concerning the problems with subject constraints mentioned earlier. By stretching/shrinking the data in this manner, we insure ourselves of minimizing the *same* loss function as in the Model III stage of Model IV.

[6]The goodness-of-fit measure utilized in all Models I–IV is defined as:

$$S^2 = \frac{\left(\sum_i \sum_j \sum_k Y_{ijk} \hat{Y}_{ijk}\right)^2}{\left(\sum_i \sum_j \sum_k Y_{ijk}^2\right)\left(\sum_i \sum_j \sum_k \hat{Y}_{ijk}^2\right)}.$$

If the data array **Y** is centered, then $S^2$ is a squared product moment correlation coefficient, and as such is interpretable as a variance-accounted-for measure (VAF). If **Y** is not centered, then $S^2$ is the square of what might be called the "uncentered correlation", and can be interpreted as a sums-of-squares-accounted-for measure (SSAF).

[7]Given the form of the model presented in equation (10), then $\alpha_i = 1/\gamma_i$. This applies the actual shrinking/stretching factor (inversely) on the left hand side (i.e., to the data). The $\gamma_i$'s are estimated in this manner rather than the $\alpha_i$'s for reasons of mathematical tractability. If instead of equation (A-18) we used:

$$Y_{ijk} \approx \alpha_i \hat{Y}_{ijk},$$

and sought least-squares estimates of $\alpha_i$, a much more complicated algorithm would be required in the succeeding stage in order to obtain new least-squares **W**, **U**, and **V** estimates.

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