Decision Issues in Building Perceptual Product Spaces with Multi-Attribute Rating Data

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This paper considers decisions that face consumer researchers as they implement a perceptual product space analysis based on multi-attribute rating data. Decisions that affect the structure of the derived perceptual product space solution can be grouped into six major categories relating to issues of (1) data input, (2) mode, (3) preprocessing transformation, (4) choice/preference modeling, (5) technique, and (6) solution. The major difficulties of each decision area are explicated, and specific recommendations are provided whenever possible.

Product space analyses based on compositional approaches that utilize brand attribute rating data have become important and frequently used tools in behavioral science research. Though comparative studies evaluating alternative compositional-based perceptual mapping techniques have recently appeared (Hauser and Koppelman 1979; Huber and Holbrook 1979), reported results and recommendations appearing in these studies are far from consensual. As such, the results have not removed the prevailing confusion concerning the similarities and differences among the various compositional approaches to building perceptual product spaces from multi-attribute rating data.[1] More important, despite the popularity of compositional approaches to building perceptual product spaces, the comparative studies published to date in the marketing and consumer behavior literature have been somewhat limited in their scope and orientation, typically focusing attention primarily on only those issues involving the choice of an appropriate technique for constructing the perceptual map. The end result is that other important but nontechnique-related issues have been virtually ignored (Hauser and Koppelman 1979; Huber and Holbrook 1979).

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In constructing perceptual product spaces, the consumer analyst must make several seemingly innocuous decisions ranging from the choice of data input to the selection of a particular perceptual mapping technique. A position unequivocably advocated throughout this article is that such decisions, although routinely made, have serious implications for the structure, meaning, and usefulness of a perceptual product space solution. Thus, because precedent all too frequently leads to convention, it is particularly important to understand the decisions that face consumer researchers as they implement a perceptual product space analysis.

The purpose of the present study is to provide an informative discussion of a number of important decision issues that typically surface in building perceptual product spaces from multi-attribute rating data. The decision issues to be discussed are organized according to the following broad categories:

1. **Data input issues**
   - On what basis should attributes be selected?
   - What is the basic form of the data array?
   - What types of data input should be used?

[1]For example, if one is to believe Hauser and Koppelman (1979), then the common factor analytic model is superior to either multiple discriminant analysis or nonmetric multidimensional scaling in terms of predictive ability, managerial interpretability, and ease of use. On the other hand, according to Huber and Holbrook (1979), principal components analysis is the preferred technique when the main concern is the linguistic relations between attributes, whereas the use of multiple discriminant analysis is recommended when the focus is on those product design attributes that can be clearly and unequivocally perceived by the consumer.

2. **Mode issues**
   - What are the effects of collapsing over respondents?
   - What are the problems involved in aggregation and the use of average correlations?
3. **Preprocessing transformation issues**
   - Should the data be normalized, standardized, or left unaltered?
   - What are the appropriate kinds of preprocessing transformations for two-way and three-way data?
   - How has preprocessing of the data been performed in perceptual product space analysis? How should it be performed?
4. **Choice/Preference modeling issues**
   - How does the choice/preference model affect the perceptual product space?
   - Which type of model is most appropriate?
5. **Technique issues**
   - What kinds of analytical techniques should be used to generate perceptual product spaces?
   - Under what circumstances should the various alternative perceptual mapping techniques give equivalent solutions?
   - How do three-mode models work?
6. **Solution issues**
   - What are the consequences of using rotation procedures in the context of perceptual product space analysis?
   - Can perceptual product spaces ever be confirmed?

In the remainder of this article, we discuss each of these basic decision issues in turn. The amount of discussion devoted to each will vary according to the importance of the issue and the extent to which the issue has been previously addressed in the multi-attribute modeling or perceptual product space analysis literature. In all cases, the discussion illuminates the problems, implications, and consequences surrounding each decision issue and, whenever possible, provides guidelines for the way in which the consumer researcher should proceed in building product spaces from multi-attribute rating data.

DATA INPUT ISSUES

The essence of prototypical perceptual mapping applications is the abstract representation of products or brands in terms of attributes that are potentially relevant to consumer choice.[2] In a reduced perceptual product space the perceived locations (i.e., coordinates or levels) of the brands define the competitive structure for the market under study. The implicit assumption is that the axes underlying the perceptual space can be interpreted in terms of specific attributes.

Attribute Selection

Except in cases where direct similarities and nonmetric multidimensional scaling (NMDS) are used, the construction of a perceptual product space typically begins with consumers rating (i.e., scaling) existing product brands and perhaps an ideal brand (and/or a set of fictitious brands) independently on the set of determinant attributes that both distinguish the product alternatives in the relevant market and reliably indicate preference or consumer choice (Alpert 1971; Myers 1970).[3] In principle, such attributes are relatable to the benefits/costs consumers seek, and may be psychological or sociological as well as physical (Shocker and Srinivasan 1979). Typically the scaling of each brand is on either semantic differential scales (Holbrook and Huber 1979; Huber and Holbrook 1979), bipolar adjectival scales (Green and Rao 1972; Johnson 1971), agree–disagree scales (Hauser and Urban 1977), or anchored scales (Hauser and Koppelman 1979; Wind 1973). Recently, Hauser and Simmie (1981) utilized a ratio-scaled paired comparison design developed by Hauser and Shugan (1980) in attempting to develop perceptual spaces that recognize the links from physical features to perceptions to preference.

Discussion. There are several issues surrounding the selection of an appropriate set of attributes that warrant some discussion. First, if bipolar or agree–disagree scales are used, then the implicit assumption is that the attributes under study fall along a continuum and that, consequently, the psychological distance between extreme positions can be divided into a fixed number of (approximately) equal intervals. Clearly, certain attributes (e.g., nominally-scaled attributes or attributes reflecting the absence/presence of a product feature) are not consistent with these scales. Second, it is important to realize that the derived perceptual space (i.e., the "dimensions") obtained by use of a data reduction procedure such as factor or components analysis is actually a function of the data collected. For example, if one attribute with high variance across brands is asked repeatedly in several different forms (e.g., subjects rate automobiles on miles per gallon, repair costs, power, etc., all of which reflect an "economy" dimension), this attribute is likely to constitute a major dimension of the derived perceptual space; in other words, uncovered dimensions may be more a function of the attributes asked than of the product features that consumers view as important.

[2]In the course of our discussion we will often use the terms "products" and "brands" interchangeably. We adopt this terminology because it is generally not known a priori whether the entries that constitute the set of alternatives are from a "conventional" product class or are drawn from different classes (Day, Shocker, and Srivastava 1978).

[3]Several alternative means are available for identifying determinant attributes (cf. Shocker and Srinivasan 1974, 1979).

Third, as extensively discussed by Shocker and Srinivasan (1974, 1979), if the consumer analyst desires *actionable product spaces,* then the attributes selected must not only be meaningful to consumers but also transformable into things that can be controlled or influenced by marketing managers.[4] Fourth, and finally, assuming that the derived perceptual product space should be related to consumer choice (or preference), additional complexities are introduced in defining the underlying attributes on which the products are scaled prior to, say, a factor, component, or discriminant analysis. In essence, the issue is whether preference is a function of the original attributes or of "latent" attributes that are uncovered in the reduced perceptual space solution. In the former case, the attributes must be scaled so that they will load on the reduced space dimensions in the same way that they influence preference judgments. In practice this is not always easily done. Shocker and Srinivasan (1979) discuss this issue further and provide an illustration that shows how the importance of the reduced space solution for preference can become relatively amorphous in the absence of the prior application of an appropriate scaling procedure.

Data Arrays

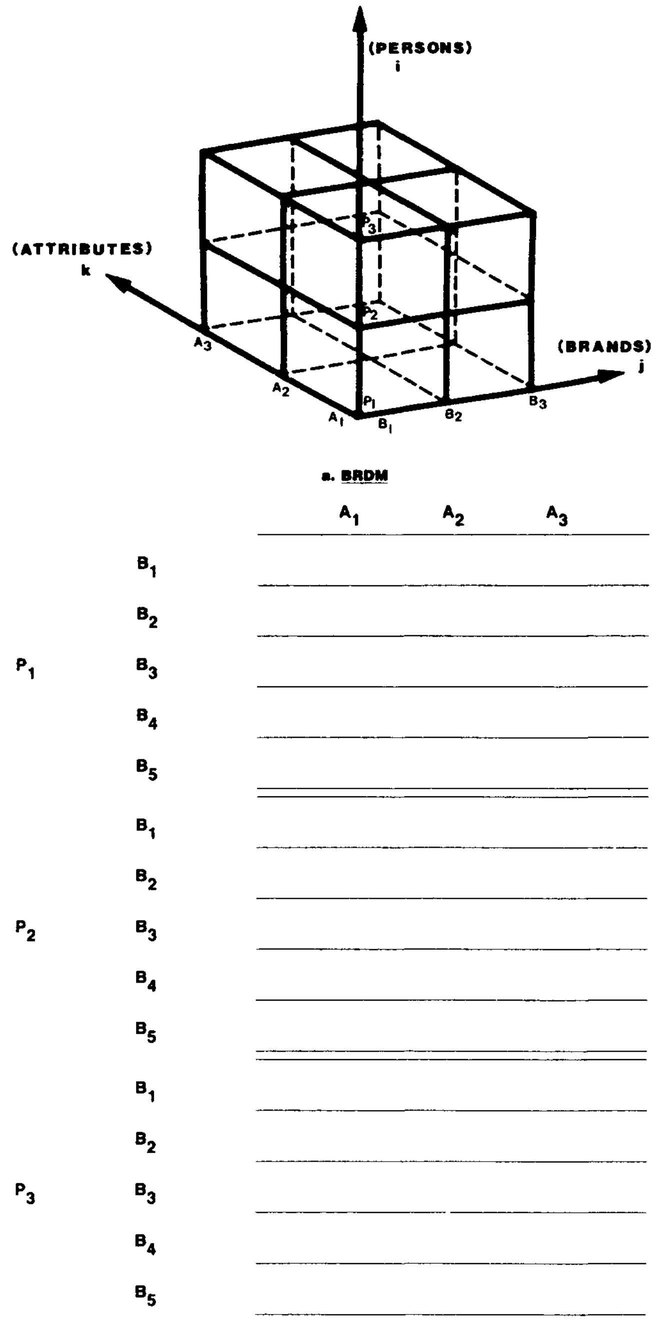
The basic input for a perceptual mapping application can be visualized in terms of a data cube or three-mode block. Specifically, a $N \times n \times p$ three-way matrix **X** is defined as a collection of elements

$$\{x_{ijk} | i = 1, 2, \ldots, N; \quad j = 1, 2, \ldots, n; \quad k = 1, 2, \ldots, p\}$$

The elements are placed in the three-mode block such that the index *i* runs along the vertical axis, the index *j* along the horizontal axis, and the index *k* along the "depth" axis. We will refer to such a collection of elements as a *3-way matrix.* Figure A shows the data cube and its two-mode representation which is formed by marking out matrices within matrices. The data cube matrix shown in Figure A is consistent with what Cattell (1952a, 1952b; 1966a, 1966b) calls the data box, or the *Basic Data Reduction Matrix* (BDRM). The BDRM, which was first called the *Covariance Chart* by Cattell (1946), systematically sets out all of the relations among the constituent components. We use the word "mode" to indicate a collection of indices by which the data can be classified. For instance, in semantic differential studies (Osgood, Suci, and Tannenbaum 1957), scores are collected on a set of bipolar *scales* for a set of *concepts* from a number of *persons.* These data can be classified by scales, concepts, and persons, and each of these determines a mode of the data. In prototypical perceptual product space applications, scores are collected from a number of *respondents* on a number of *brands* for a set of *product features* or *attributes;* therefore, the modes of the data are in terms of respondents, brands, and attributes (see Figure A).

FIGURE A

BRDM AND 3-WAY MATRIX



[4] This problem is mitigated if the purpose of the analysis is only concept evaluation (cf. Shocker and Srinivasan 1979).

Discussion. Although it is customary in perceptual product space applications to analyze three-mode data classified according to respondents, brands, and attributes, there is no logical requirement that the 3-way matrix always include only these specific components. Recent research in marketing and social psychology, for instance, has shown that situational factors (i.e., usage context or intended usage) are important moderators of consumer behavior (cf. Belk 1974; Berkowitz, Ginter, and Talarzyk 1975; Calatone and Sawyer 1978; Hustad, Mayer, and Whipple 1975; Miller and Ginter 1979; Srivasata, Shocker, and Day 1978). Thus it is reasonable to expect that data classified according to, say, respondents, brands, and usage context preference be collected for perceptual product space analysis. For that matter, other high order data matrices of four or more modes might exist. For example, a consumer researcher could have a sample of respondents indicate their perceptions of a number of different fast food establishments (McDonald's, Burger King, Wendy's, etc.) on a number of attribute rating scales (convenience, service, quality, etc.) for a diverse set of consumption situations (breakfast, lunch, dinner, etc.). In such a consumer behavior study a respondent × establishment × attribute × situation four-way matrix would be collected, and interest would center on the relationship among the various components as well as on their separate effects. Recently, Lastovicka (1981) presented such an analysis in which the four-mode data consisted of 27 respondents, 6 different television advertisements, 5 separate exposure occasions, and 16 reaction items.

Type of Data Input

Perceptual mapping techniques such as factor analysis (FA) and principal component analysis (PCA) could conceivably be applied directly to the raw data or indirectly to either summed cross-products, covariances, or correlations (Kruskal 1978).[5] Correlations are a special case of covariances (i.e., the data for each variable have unit variance). Covariances are, in turn, a special case of summed cross-products (i.e., the data for each variable have zero mean), and summed cross-products are formed by performing a simple matrix operation on the raw data matrix (i.e., **X′X**). Both FA and PCA provide reparameterizations of the original observations, where the differences between these two representations depends on the assumptions concerning the contribution of unique variance. In the case of two-mode data the structure underlying a data point x_{ij} can be represented by

$$x_{ij} = \sum_{r=1}^{q} (a_{ir} f_{jr}) + e_{ij} \quad (1)$$

or in matrix form as

$$\mathbf{X} = \mathbf{\Lambda F'} + \mathbf{E}, \quad (2)$$

where **X** is a two-way matrix with p rows and n columns. We can think of the rows as representing attributes and the columns as representing entries. For a model in terms of q factors, **Λ** is a pxq matrix of factor weights on variables, sometimes called "loadings," and **F** is a nxq matrix of factor weights on cases, sometimes called "factor scores." These weights reflect the degree to which the factor is expressed in the particular attribute or case. **E** represents a matrix of random error terms, sometimes called "unique effects." Note that if we assume no unique effects, then the factor analysis model shown above reduces to a PCA specification. In either case, given the specification, it would appear natural to obtain factor loadings and factor scores by directly analyzing the raw data matrix. Indeed, some analysts (Horst 1965; Kruskal 1978) have argued that the direct fitting approach is the most mathematically appropriate and logical method of estimation. However, from an historical perspective, FA and PCA have typically been applied to correlations rather than to the raw data. This practice continues to be the standard convention even to the present day.

Discussion. One reason for the dominance of correlational input is that it reduces the computational effort required. If the data array is two-mode, say, representing attribute × brand ratings or respondent × brand ratings, it turns out that the solution based on the correlation matrix is identical to the solution obtained from the standardized (i.e., *z*-score) data themselves. If the data array is three-mode, however, this equivalence no longer holds, and correlation input should generally be avoided. The reasons for this are technical. Harshman and Lundy (1984a) provide an excellent discussion of this issue. Essentially, correlations are inappropriate as input for any currently available three-mode data analysis procedure, including PARAFAC (Harshman 1976), CANDECOMP (Carroll and Chang 1970), and the Tucker models (Tucker 1966, 1972). Because by computing correlation matrices for each (three-way) slice of the original data array separate scalings on each attribute for each slice are imposed, the respective factor loading matrices are modified in a complex way; it is difficult, if not impossible, to assess the true contribution of an attribute to a factor across the various (three-way) slices (see Harshman and Lundy 1984a, pp. 133–141 for further details).[6] We will have more to say about the use of correlations in the section on dimensionality issues which immediately follows.

[5]Alternatively, distances can also be analyzed. We discuss this form of data input in a later section of the article.

[6]Jöreskog (1971) also points out that correlations are not appropriate in his across-population factor analysis models.

MODE ISSUES

Because conventional perceptual mapping techniques, such as FA and PCA, are only designed to analyze one slice of the data matrix at a time, solutions that are used to construct the perceptual product space are based on some sort of reduction of the data to two modes. The usual practice is to either collapse or average over respondents and analyze the two-mode brand × attribute data.

Effects on the Perceptual Space

The results of a perceptual product space analysis are frequently summarized by a perceptual map that shows the location of each brand in the reduced space. If FA or PCA is used to generate the perceptual space, brands are positioned on the basis of their factor scores, and the latent dimensions (i.e., factors) are interpreted by examining factor loadings that are estimates of the correlations between attribute ratings and the uncovered latent dimensions.

Discussion. An obvious liability of conducting a brand × attribute analysis by collapsing over respondents is the possibility that respondents have different perceptual processes; that is, this approach assumes that the perceptual processes of the respondents are similar and does not allow for individual differences in structure. The effects of this type of aggregation can be severe; we will discuss them further in a later section.

There are, however, other more subtle issues involved in the asymmetrical treatment of the data that characterizes most reduction procedures. First, since the focus of this kind of two-way analysis is on the relationships among attributes across the modes making up the respondents and the brands, the implication is that differences in means and standard deviations introduced by the different brands are not of interest. However, as we discuss below, such differences in means or standard deviations introduced by the brands will affect the interattribute correlations in complex ways; moreover, information about mean and standard deviation differences among brands (and, for that matter, respondents) would appear to be particularly important in perceptual product space analysis.

Second, the asymmetrical treatment of the data limits the types of perceptual spaces that can be investigated. In prototypical product space analysis the factors obtained are dimensions along which attributes may differ. Because the data are treated asymmetrically, we have a factor loading matrix only for the attributes, and the brands are positioned in the perceptual space on the basis of their factor scores. Thus, we do not have a simultaneous *joint space* in the strict sense of a *classical multidimensional unfolding* solution (Coombs 1964); ideally, we would like to construct a perceptual space based on a simultaneous analysis of both modes of the data wherein the brand factors and the attribute factors are jointly represented—in other words, wherein both brands and attributes are reduced to factors where the brand factors and the attribute factors can be related. Levin (1965) presents a modification of conventional two-mode factor analysis that treats the rows and columns of a two-way matrix symmetrically.

Third, and finally, it is conceivable that other types of perceptual maps that position, for example, brands and respondents or brands, attributes, and respondents in the space may be informative. Conventional two-mode analysis will not prove totally satisfactory because even if independent factor analyses are performed on each slice of the data, problems in interpretation are likely to arise, since there may be no easy way to relate the various solutions because of differences in rotation; moreover, the separate analyses are not strictly independent.

Aggregation Problems

In uncovering the latent dimensions that define the reduced space, inter-attribute correlations are typically derived by stacking the brand-by-attribute subarrays for all the respondents into one long matrix, with columns representing each attribute and rows representing each respondent's rating of each brand (cf. Hauser and Koppelman 1979; Huber and Holbrook 1979); thus the interattribute correlations are computed across brands and respondents. We will refer to this practice as the *extended data matrix* approach.

Discussion. The use of the extended data matrix approach in which correlations are computed across an entire sample (i.e., respondents and brands) can produce quite misleading results. There are two primary reasons for this. First, the correlation between any two attribute characteristics computed across an entire sample is actually a function of two different sources. One source contributing to the correlation relates to the consistency or inconsistency of responses to the set of attribute characteristics. Another relates to how homogeneous or diverse the respondents are in their attribute rating judgments. To illustrate, suppose that in a sample it is possible to identify three clusters of respondents. Suppose, further, that each of the three groups is (1) relatively consistent (i.e., either a high-high pattern or a low-low one) in its brand ratings on all of the attribute characteristics, and (2) relatively consistent in its brand ratings between the attribute characteristics. Condition 1 implies—say, because of common prior experiences or background factors—that the range of variation on the attribute characteristics within each group is small relative to the variation between the respective groups. Condition 2 implies—say, because of halo effects or the conceptual similarity of the attribute characteristics—that respondents evaluate brands consistently on pairs of attributes. Thus, we might say that members of the respective groups exhibit highly structured or consistent attribute perceptions.

Figure B depicts this situation. Note that the distribution of respondents takes the form of three relatively tight clusters. Each dot represents a particular respondent's rating of a brand on each of two attribute characteristics. Group A is clustered at low values on both attributes with mean $\bar{X}_{A1}$ and $\bar{X}_{A2}$, group B is clustered at medium values on both attributes with means $\bar{X}_{B1}$ and $\bar{X}_{B2}$, and group C is clustered at high values on both attributes with means $\bar{X}_{C1}$ and $\bar{X}_{C2}$. Thus we see that the correlation between attributes is actually due to the mixing of heterogeneous groups, and that within a group the attribute ratings are uncorrelated. This is not so unrealistic an assumption as might first appear, since it implies that within a perceptually homogeneous group, variation of one attribute is more likely idiosyncratic and does not represent the effect of any systematic tendency to be higher or lower on the other attribute.

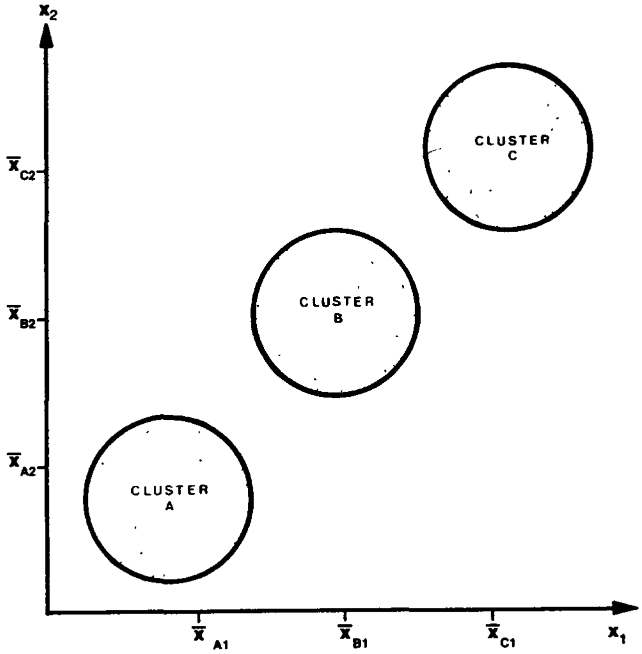
Actually, readers familiar with *latent structure analysis* (cf. Lazarsfeld and Henry 1968) will recognize the zero correlation within the attitudinally homogeneous groups as the "axiom of local independence." Moreover, from the perspective of measuring belief system structures, Barton and Parsons (1977) demonstrate how comparison based upon item intercorrelations must be misleading if the populations being compared are not equally heterogeneous. They show that the value of the correlation coefficient is a function of the distance between the groups and the relative variance within each group, and provide empirical evidence to support the veracity of their contentions.

The extended data matrix approach can also produce misleading results because it pools attribute ratings across respondents and brands and consequently produces what is essentially an "average" correlation coefficient. Such average correlations can be acutely affected by mean differences introduced by the brands or, for that matter, by "outliers" or untypical responses. For example, attributes that do not correlate at all for a given brand can have high correlations if they have the same large mean differences across the brands; on the other hand, two attributes that are highly correlated but have means that are affected differentially, depending on the brand, may have that correlation wiped out. (In a later section of the article we discuss how a preprocessing transformation of the data can dampen these effects.)

There is also empirical evidence to suggest that pooling attribute ratings across respondents and brands is not to be recommended. Calatone and Sawyer (1978), for example, hypothesized that the instability of cluster solutions was due to the situation-specific weights assigned to the importance of product features and to the variation in individuals' usage situations over time. Miller and Ginter (1979) argue for the recognition and use of situation-specific measures in attribute-based attitude models. Finally, in the context of a perceptual product space analysis, Hustad, Mayer, and Whipple (1975) have demonstrated that consumer ratings of beverage attributes and ideal beverages were dependent on the usage context imposed. Thus, there is conceptual, mathematical, and empirical evidence that calls into question the common practice of reporting aggregate perceptual product space solutions that do not retain differences due to individuals, brands, or usage contexts.

FIGURE B

AN ILLUSTRATION OF THE ADVERSE EFFECTS OF HETEROGENEOUS GROUPS ON THE CORRELATION COEFFICIENT



PREPROCESSING TRANSFORMATION ISSUES

Preprocessing the data refers to the practice of performing any transformation of the data values prior to applying the technique that will be used to generate the perceptual product space. Harshman and Lundy (1984b) discuss many reasons for preprocessing data before performing a perceptual product space analysis. Among the more important reasons are: (1) to make the data congenial with the assumptions underlying the model used to generate the perceptual product space, (2) to eliminate various unwanted sources of variation from the final solution, (3) to emphasize certain relationships in the final solution, (4) to facilitate a comparison of factor loadings across levels, (5) to facilitate comparisons across data sets,

EXHIBIT 1

SIX PRELIMINARY DATA TRANSFORMATIONS USED IN FACTOR ANALYSIS

Transformation	Description	Consequences
Normalization	Transforms each column vector of the data matrix by dividing each element by the square root of the sum of the squared elements. Normalizing the variable scales the data so that the sum of their squares equals unity.	Scaling a data matrix by normalizing the column vectors contracts or lengthens the vectors in space so that they all have a magnitude of 1—a unit length. If the data units from vector (variable) to vector (variable) are non-comparable, normalizing the vectors will make their data comparable. It does so by equating vector magnitudes, but without equating the means and standard deviations between the vectors.
Standardization	Subtracts the mean of the data for a variable from the original data and then divides by the standard deviation.	The effect of standardization is to remove the difference in means and deviation between variables from their covariance. The data are reduced to common units of deviation around the mean—standard score units. This allows comparison not only of data that have different units, but also of data on different measurement scales.
Centering	Centers the variables or observations of a data matrix by subtracting their respective means. If all the variables are centered, the columns of the scaled matrix will sum to zero; if all the observations are centered, the rows of the scaled matrix will sum to zero.	The effect of centering is to remove the covariance associated with the different deviations around the means. Since standardization removes covariance associated with both mean and deviation differences, centering retains more information in the data than does standardization. Centering transformations, however, do not retain as much information as does the normalization transformation, which merely contracts the column vectors to unit length.
Ipsative scaling	Any transformation that produces a scaled matrix in which the rows of the matrix sum to the same value is said to be *ipsative* (cf. Cattell 1944). Thus an ipsative scaled matrix can result from either a standardization or centering of the rows of the original data matrix. A comprehensive treatment of ipsative measures is given by Clemans (1956).	Only intraindividual comparisons are meaningful; interindividual comparisons are not. Centering the rows of a matrix reduces the rank of a matrix by one (cf. Horst 1965, p. 294) when the number of variables is less than or equal to the number of cases.
Bounding	Scales the range of values within a matrix to be within certain bounds. This is done by similarly increasing or decreasing the magnitude of the data without equating the data's different means and standard deviations.	Transformations of this kind may allow easier interpretation of the factors resulting from a direct factor analysis or produce data that satisfy certain mathematical requirements of the factor techniques involved.
Null scaling	Rescales the data to deviations from expected (null) values. The matrix is thus transformed according to a null statistical model so that significant deviations from the null model can be ascertained.	A null matrix transformation can alter the interrelationships identified by the factor analysis. For a null-scaled matrix, the factors will identify those variables that cluster in terms of large deviations of their values from expectation.

and (6) to increase the ease of interpreting the final solution. "Good" preprocessing will accomplish the desired objective(s) while leaving the latent factors relatively unchanged.

Preprocessing in Two-Mode Factor Analysis Models

Many different types of preprocessing transformations have been utilized, albeit amorphously, in the factor analysis literature. Rummel (1970, Ch. 12) describes six preliminary data transformations: *normalizing, standardizing, centering, ipsative-normative scaling, boundary scaling,* and *null scaling.* Centering and ipsative-normative scaling are *additive adjustment* transformations, whereas normalizing and standardizing are *multiplicative adjustments;* boundary scaling and null scaling represent more specialized transformations and have not been used as frequently as the other four preprocessing transformations. Exhibit 1 describes each of the preprocessing transformations and indicates their consequences.

Discussion. The preprocessing transformations described in Exhibit 1 presume at least interval scale data. The issue of the strength of measurement is important, since the lack of strong measurement is often a reason for effecting a preprocessing transformation and subsequently using correlational input with traditional two-mode factor analysis models. Traditional two-mode factor analysis models are formulated in terms of ratio scale quantities which, of course, presume knowledge of the true zero-point on the variables from which the factors are to be extracted. However, factor analysts have handled interval scale data without much difficulty. In large measure, this is because the factor analysis of two-mode data has been conceptualized in terms of correlation matrices. The conversion of the raw data to correlations, which involves both additive (centering) and multiplicative (standardizing) transformations, eliminates, by the centering operation, the problem of origin (Harshman and Lundy 1984b).

All of the preprocessing transformations produce data that retain varying amounts of the original (scale) information. The effects of alternative preprocessing transformations can be understood by examining a simple model of the raw data that includes both the factors that we wish to extract plus the unwanted components that obscure the true origin and make the data interval-scale rather than ratio-scale. Following Harshman and Lundy (1984b) let x_{ij} be the element in the i^{th} row and j^{th} column of X, and consider the following representation:

$$x_{ij} = \sum_{r=1}^{q} (a_{ir}f_{jr}) + h_i + h_j + h + e_{ij} \qquad (3)$$

The a_{ir} and f_{jr} terms represent the factor loadings and factor scores for modes A and B, respectively; e_{ij} represents random error, and the additional h_i, h_j and h terms represent the unknown constants that disturb the proportionality and cause the data to have interval-scale rather than ratio-scale properties. The interpretation of these unknown constants is straightforward; for example, we can view the h term as an overall additive constant that offsets the data as a whole from true ratio scale properties (i.e., it shifts the zero point along a scale). Because of respondent-to-respondent variation, an h_j term is also needed that describes how the j^{th} column deviates from a ratio scale, after adjusting by h (i.e., it specifies the shifts in the baseline for the j^{th} rater). Finally, an h_i term is needed that describes how the i^{th} row deviates from a ratio scale after adjusting by h (i.e., it specifies the shifts from row to row).

Factor analyzing the raw data without any preprocessing and retaining only q factors produce misleading results since the loadings would be distorted by the presence of the constants h_i, h_j, and h in the data. Additive preprocessing transformations eliminate these constants. Column centering, for example, will eliminate h_j (as well as h), whereas ipsative scaling will remove h_i (as well as h). As a biproduct of centering across any mode of the data, the factor loading matrix for that mode will be column centered. Multiplicative transformations such as normalization or standardization will also eliminate these constants; in addition, such transformations can handle noncomparable interval scales having idiosyncratic origins and units of measurement. In general, the choice of a particular preprocessing transformation depends upon, among other things, the data units, the measurement scales, the slice of data under study, the use of correlation or covariance measures, and the extent to which across-factor comparisons are to be made. However, since the primary reason for using a preprocessing transformation is to remove unwanted constants that disturb the ratio scale properties of the data, in most applications a simple centering by rows and columns will eliminate any unwanted components while preserving the factor structure underlying the data (Harshman and Lundy 1984b).

Though with two-mode data the effects of different centering operations are straightforward, there has been considerable controversy on this point (Burt 1937; Cattell 1944, 1952a; Gower 1966; MacAndrew and Forgy 1963; Ross 1963). The argument centers on whether different factors can be uncovered by different centering operations. As Harshman and Lundy (1984b, pp. 229–230) discuss, one reason why some investigators have believed that different centerings will produce different factors is that the different centerings can alter the relative emphasis of several dimensions underlying a data set. The relative emphasis of a dimension before and after centering is a function of the change in the sums-of-squares of the factor loadings. Changes in the sums-of-squares of the factor loadings will change the relative contribution of the dimensions to each unrotated factor or principal component. If, in order to obtain a lower-order solution, some non-negligible components are discarded, then the centered and uncentered solutions may be different depending on whether different parts of the factor space have been thrown away. Additional misconceptions can be traced to the rotational indeterminacy of the traditional two-mode factor analysis model. Centering will change the orientation of the principal components and will also change the best simple structure orientation. These differences are illusionary since, in general, for any given axis orientation, a column-centered factor matrix produces the same factor interpretation as an uncentered factor matrix.

To summarize, then, the consequences of centering the data are simple: the factor loadings for one mode are centered, and the unwanted constants disappear. The centered data is now ratio scaled and can be represented (aside from the random error) simply in terms of the underlying factors. We can recover these factors accurately after the centering operation, since the factor score estimates are unaffected by the centering, and the factor loadings are the same except that mean values for each factor have been subtracted out (that is, the loading matrix is column-centered). Thus the shape of the profile of factor loadings is unaffected, and only the elevation is changed. Since most of the interpretation of a factor is based on the relative values of the loadings on the different stimuli rather than on the absolute size of the loadings, the centering operation does not normally interfere with our interpretation of the factor.

Processing Three-Mode Data

Because of the many alternatives that exist, the question of proper preprocessing of a three-way matrix becomes more complex. Harshman and Lundy (1984b) have presented an excellent comprehensive treatment of preprocessing transformations for three-mode data. The various alternative additive and multiplicative preprocessing transformations of three-mode data can be discussed in terms of whether a one-way, two-way, or three-way transformation is performed. For example, one-way centering is accomplished by subtracting means computed over one-way subarrays from the raw data; i.e., $x^*_{ijk} = x_{ijk} - x_{.jk}$ represents an across-Mode A one-way centering operation. Two-way centering is accomplished by subtracting means computed over two-way subarrays from the new data; i.e., $x^*_{ijk} = x_{ijk} - x_{..k}$ represents an across-Modes A and B two-way centering operation. Finally, three-way centering is accomplished by subtracting a mean computed over the entire three-way array from the raw data; i.e., $x^*_{ijk} = x_{ijk} - x_{...}$.

Discussion. In large measure, the complexities introduced by three-mode data are due to the interactions between the respective modes. Three-way models—e.g., PARAFAC (Harshman 1970) or Tucker's three-mode models (Tucker 1966)—focus on components whose pattern of change for levels of one mode depends simultaneously on the levels of the other two modes involved (i.e., three-way interactions in the usual analysis of variance). For example, given an attribute × brand × respondent array, to take all the unwanted constants into account we must consider the consequences of not only the global constant h and "one-way" effects, h_i, h_j, and now, in addition, h_k, which are constant across two modes, but also the consequences of the "two-way" effects, h_{ij}, h_{ik}, and h_{jk}, which are due to interactions over a third mode. The basic three-mode representation is

$$x_{ijk} = \sum_{r=1}^{q} (a_{ir} f_{jr} c_{kr}) + h_{ij} + h_{ik} + h_{jk} + h_i + h_j + h_k + h + e_{ijk}, \quad (4)$$

where the a_{ir}, f_{jr}, and c_{kr} terms can be interpreted as factor loadings for each of the Modes A, B, and C, respectively, e_{ijk} represent random errors, and the various h terms represent the unwanted component effects. Thus, "good" preprocessing with three-mode data means that the unwanted components of the data h_{ij}, h_{ik}, h_{jk}, h_i, h_j, h_k, and h are eliminated while preserving as much as possible the original structure underlying the data.

Harshman and Lundy (1984b) present a detailed discussion and sundry proofs concerning the effects of performing one-way, two-way, and three-way preprocessing transformations. Interestingly, two-way centering transformations, which appear to be the most logical and natural for three-mode data, are not recommended. Two-way centering is undesirable for several reasons: (1) it does not remove two-way effects—i.e., h_{ij}, h_{ik}, or h_{jk}; (2) it introduces unwanted constants; and (3) it removes only one of the one-way constants—i.e., either h_i, h_j, or h_k (Harshman and Lundy 1984b, p. 238).

A number of practical guidelines for selecting a preprocessing transformation suitable for three-mode data can be found in Harshman and Lundy (1984b, pp. 257–258). Several of their recommendations and findings are relevant to our discussion of preprocessing transformations in perceptual product space analysis that immediately follows. Among these are:

1. It is generally advisable to remove meaningless arbitrary differences in scale size across a given mode due to perhaps differences in the unit of measurement by standardizing within the levels of that mode.
2. With three-way rating scale data, for example, stimuli × scales × persons, good results have been obtained by centering the stimuli and size-standardizing the scales and the persons.
3. In general, standardizing two modes and centering two modes, but not the same two modes—i.e., one mode is simply centered, one mode is simply standardized and one mode is both centered and standardized—has produced good results.
4. Frequently, several different preprocessing transformations will have to be used and evaluated in terms of their effects on the factor analytic solutions that result.

Preprocessing in Perceptual Product Space Analysis

Not surprisingly, preprocessing transformations have been frequently used in perceptual product space analysis. However, the implications of having three-mode attribute × brand × respondent data are frequently ignored, and there is generally little discussion of the reasons for or consequences of effecting a particular kind of preprocessing transformation. The most frequently cited reason for undertaking a preprocessing transformation is to remove "scale bias" and "yea-saying" effects (cf. Hauser and Koppelman 1979; Holbrook and Huber 1979; Huber and Holbrook 1979). Although not discussed in this context, these effects presumably reflect unwanted constants—i.e., h_i, h_j, h_k, and h terms. Three types of preprocessing

EXHIBIT 2

THREE PRELIMINARY DATA TRANSFORMATIONS USED IN PERCEPTUAL MAPPING APPLICATIONS

Transformation	Description	Consequences
Standardization by individual	Standardization by individual produces within-respondent standard scores. The standardized score for the i^{th} respondent is given by $(X_{ijk} - \bar{X}_{ij\cdot})/s_{ij\cdot}$ where $s_{ij\cdot}$ is the standard deviation of the i^{th} respondents' scores computed by summing over attributes for each brand.	Row order is meaningful, but column order is meaningless as far as the absolute strength of a given attribute is concerned. It is not permissible to compare the ranking for different respondents for the same or different brands within the column. In the case of a doubly standardized matrix, variation in the data matrix associated with the main effects of rows and columns is removed. What remains is the variation due to row × column interaction only. Ipsative correlation matrices do not belong to the basic or nonsingular class. The scaled, transformed, doubly standardized matrix is not of full rank and thus one or more of the eigenvalues induced from factoring the resulting ipsative correlation matrix will be zero.
Standardization across respondents and brands	Standardization across respondents and brands involves replacing the i^{th} respondents' original score by $(X_{ijk} - \bar{X}_{\cdot\cdot k})/s_{\cdot\cdot k}$ where $s_{\cdot\cdot k}$ is the standard deviation computed by summing over respondents and brands for each attribute.	Both column and row comparisons are meaningful, though a certain amount of information may be lost by equating means and variances.
Normalization	This transformation is accomplished by dividing each attribute rating score by the sum of the ratings across the brands for a given respondent and attribute; that is, the i^{th} respondent's original score is replaced by $X_{ijk}/\sum_j X_{i\cdot k}$	Column order is meaningless and row comparisons are permissible for different brands for the same respondent. The scaled, transformed correlation matrix will not be equivalent to the original correlation matrix, although the discrepancy in eigenstructures should be relatively minor.

transformations have been popular in perceptual product space analysis: *standardization by individual* (Hauser and Koppelman 1979), *standardization across respondents and brands* (Green and Rao 1972), and *normalization within individual* (Huber and Holbrook 1979).

As we indicated earlier, the convention in perceptual product space literature is to view the three-way matrix in terms of a two-way extended matrix in which the brand-by-attribute subarrays for all the respondents are stacked into one long matrix, with columns representing each attribute and rows representing each respondent's rating of each brand. In the ensuing discussion we will assume that the basic data matrix has this form. Standardizing across respondents and brands (i.e., over the rows of the data matrix) is what we do when interattribute correlations are computed. Normalization within individual is accomplished by dividing each attribute rating by the sum of the ratings across brands for a given respondent, with the result that each respondent's rating on an attribute sums to unity across brands. Standardization by individual involves a standardization across attributes for each respondent's ratings of each brand. This type of within-respondent transformation produces what is called an *ipsative scale matrix.* With *fully ipsative scaling,* a respondent's score is evaluated relative to his/her own mean and variance of all attributes. Exhibit 2 describes each of the preprocessing transformations and indicates their consequences.

Discussion. At the most basic level of analysis, preprocessing transformations affect the types of comparisons that can be meaingfully undertaken. The appropriateness of different types of comparisons is complicated by the three-mode nature of the data. For discussion purposes, consider column comparisons as those between the respondent's rating of a brand on a given attribute, and row comparisons as those between attributes. It can be easily verified that the following restrictions on the permissible set of comparisons hold:

1. When the scores are in raw units, column comparisons are meaningful but row comparisons are meaningless.
2. In standardizing across respondents and brands for a given attribute, both column and row comparisons are meaningful, though a certain amount of information may be lost by equating means and variances.
3. With data standardized by respondent across attributes for each brand, the row order is meaningful, but column order is meaningless, as far as the

absolute strength of a given attribute is concerned; in addition, it is not permissible to compare the ranking for different respondents for the same or different brands within the column. (This is because a respondent's score is evaluated relative to his/her mean and variance on all attributes.)

4. In normalizing the raw scores within respondent across brands for a given attribute, column order is meaningless and row comparisons are permissible for different brands for the same respondent.

Preprocessing transformations can have analytical consequences. For example, standardizing the raw scores across respondents and brands for each attribute does not alter in any way the original correlation matrix. In this case, the standardization is along the series being correlated and the resulting correlation structure of the scaled transformed matrix is consequently unaffected. The eigenstructure of the raw correlation matrix will, of course, be equivalent to that of the preprocessed correlation matrix, simply because the two input matrices are identical.

Scaling a data matrix by normalizing the column vector (i.e., attributes or variables) such that a respondent's rating on an attribute sums to unity across the brands contracts (or lengthens) each respondent's attribute vector in space to unit length. Normalized scale transformations retain a maximum amount of scale information from the original data. In general, normalizing the entire column vector of attribute ratings will make the data comparable by equating vector magnitudes without equating the means or variances between the vectors; thus the scaled transformed correlation matrix and the original correlation matrix will have the same basic structure. For the type of normalized scale transformation undertaken in the context of generating perceptual product spaces, this is not the case, however, since the normalization is accomplished across brands for a given respondent. In this situation, the scaled, transformed correlation matrix will not be equivalent to the original correlation matrix.

Within-respondent standardization across attributes for each brand involves a transformation that is across the direction of correlation; that is, it standardizes the rows before correlating columns. As we indicated, this type of scale transformation produces what is called an ipsative scale matrix. If an ipsative scale matrix is transformed to a correlation matrix, another standardization is undertaken—this time with respect to the columns of the matrix, which means that the resulting ipsative correlation matrix is *doubly standardized.*[7] In the process of doubly standardizing a data matrix, the matrix is doubly centered. In essence, this preprocessing transformation removes the row and column mean effects (i.e., elevation) so that the centroid of the row and column configuration is shifted to the origin of the component space. After double centering, the variation that remains is due to row and column interaction (Gabriel 1978; Gollob 1968). The effect of double centering a matrix on the factor solution is to make the mean of each component column in the respective factor loading matrix equal to zero. Thus, the factors highlight individual differences—that is, the effects of factors are interpreted as deviations of individuals from the mean measures; consequently, the mean measure is considered as a basic characteristic of the responses of the respondents that is used as a base from which to describe individual differences in the pattern of responses.

In contrast, Tucker (1968) contends that the analysis should be performed on the original measures with no special a priori status given to the mean measure. In Tucker's view, general tendencies and individual differences should be considered simultaneously since the purpose of doing an analysis is to describe factors that are important in determining the original measures and not the row × column interaction. In cases where individual differences are of primary interest and ipsative scale matrices are analyzed, it is important to note that ipsative correlation matrices do not belong to the basic or nonsingular class. Because the scaled, transformed, doubly standardized matrix is not of full rank, one or more of the eigenvalues induced from factoring the resulting ipsative correlation matrix will be zero. A comprehensive treatment of the properties and problems imposed by ipsative scores is given by Clemans (1956).

Because of the three-mode nature of the raw data a pre-processing transformation is required if conventional two-way perceptual mapping techniques are used. In conventional perceptual product space analysis, the raw data matrix has nN rows where the first n rows refer to the ratings of the n brands for the first respondent, the second n rows to the second respondent, and so on for all N respondents. The problem is that if this matrix is analyzed as is, the variance in the attribute ratings for the respondents is confounded with the variance of the brands on an attribute for each respondent. Stated differently, the between-respondent variation on attributes will confound the within-respondent variances on brands.

A standardization of the raw data can help, however. Specifically, we recommend that to bring out the within-respondent variance on the attribute characteristics over the brands for each respondent and to allow generalizations across brands, each attribute should be standardized across the brands for each respondent separately. Thus, data on the first attribute for the first respondent will be standardized over the n brands,

[7]Doubly standardizing a matrix will involve an iterative process. Standardizing rows after standardizing columns upsets the column standardization so that columns have to be restandardized. Restandardization will then upset the row standardization. One has to work back and forth between column and row standardization until the standard deviations for columns and rows converge within some acceptable limits.

then the data for the same attribute for the second respondent will be separately standardized, and so on for all respondents on this and every other attribute. These separate, within-respondent standardizations provide a reasonable approach to collapsing the three-mode data to a two-way matrix. The standardizations within each respondent will remove from the analysis the variance on the attribute characteristics due to individual level effects. The remaining variance gives the interrelationships between changes in the attribute characteristics over the brands generalized across the respondents. This type of preprocessing transformation of the raw three-mode data is consistent, at least in spirit, with the practical guidelines suggested by Harshman and Lundy (1984b) discussed in the previous section.

CHOICE/PREFERENCE MODELING ISSUES

One of several versions of what is termed a compensatory model (Green and Srinivasan 1978; Green and Wind 1973) is typically used to model consumer decision making. Two of the more popular versions of the compensatory models are the *ideal point version* and the *vector version.* Because excellent discussions of these models have appeared in the consumer behavior and marketing literature (Green and Srinivasan 1978; Shocker and Srinivasan 1974, 1979) our remarks will be brief.

Ideal Point Model. Underlying the ideal point model is the assumption that some amount of product attribute is ideal. The respondent's most preferred combination of attributes is the ideal point, and the closer a given brand is to the respondent's ideal point, the greater the probability that the brand will be chosen (i.e., preferred over other brands farther away from the ideal point). Thus, the tacit assumption is that the respondent's utility for a given brand is inversely related to its weighted Euclidean distance from his/her ideal point. DeSarbo and Carroll (1981) have presented an ideal point model that can accommodate three-mode data.

Vector Model. Underlying the vector model is the assumption that the more (less) the better. A respondent's utility is modeled directly as a weighted sum of the attribute levels of each (brand) alternative. We can view the vector version as a special case of the ideal point version by considering the ideal levels for every attribute as being plus or minus infinity so that the respondent must by necessity always prefer more (less) of each attribute.

Discussion. The decision as to whether to use a vector model or an ideal point model will largely depend on the alternatives being compared. If the set of alternatives does not contain brands which have either too much or too little of each attribute, then the vector model will be most useful. For example, if respondents are rating brands of diet beverages that vary in carbonation and sweetness—but none of the alternative brands are carbonated enough or sweet enough for their tastes—then the vector model is the appropriate model of preference since more is better. On the other hand, if the set of alternatives does contain brands which have either too much or too little of at least one attribute, then the ideal point model is the most useful. For example, suppose that in a diet beverage study the alternatives do in fact vary from exceedingly sweet to nonsweet; in such a case, it is probably true that there is some ideal sweetness for each respondent. Finally, it is important to recognize that since both models are defensible, mixed models in which one (or more) attribute is always preferred in greater (lesser) amounts (vector) while other attributes are preferred in moderate levels (ideal point) may have empirical advantages for the general case (Shocker and Srinivasan 1974, 1979).

The importance of considering the choice/preference model to which the ultimate perceptual product space is connected cannot be overemphasized. Suppose that in the diet beverage study respondents prefer more carbonation but prefer moderate levels of sweetness, bitterness, sharpness, and strength. Suppose further that all of the attributes influence consumer choice but that carbonation is the most important. If the carbonation attribute is relatively independent of the rest, then depending on the method of constructing the perceptual product space, the carbonation attribute may be overlooked; in other words, it can happen that an attribute that is most important in determining consumer choice behavior could be overlooked if it is relatively independent of other attributes. We return to this issue later when discussing the relationships among the various approaches to building perceptual product spaces based on multi-attribute rating data.

TECHNIQUE-RELATED ISSUES

There are two broad classes of techniques suitable for compositional-based perceptual product space analysis: two-way analysis techniques and three-mode models. Three-mode models have only recently appeared and have been primarily applied in the analysis of semantic differential data involving scale × concept × respondent interactions (Gitin 1970; Hentschel and Klintman 1974; Muthen et al. 1977; Snyder and Wiggins 1970; Tzeng 1976). Though three-mode models are relatively new and have not as yet been extensively applied in perceptual product space analysis, we do discuss them briefly in a later section.

Two-Way Analysis Techniques

The four major techniques for generating compositional-based perceptual product spaces with multi-attribute rating data are factor analysis (FA), principal components analysis (PCA), ratio multidimensional scaling (RMDS), and multiple discriminant analysis (MDA). Few direct comparisons of these methods have been made; however, in those that have been published the emphasis is on empirical analysis in which the different methods are applied to a single data set to highlight their differences (Hauser and Koppelman 1979; Holbrook and Huber 1979; Huber and Holbrook 1979).

Discussion. Apparently there is good reason to believe that these methods will generate different product space solutions since the solutions are based on extracting the underlying dimensions from matrices of different orders. FA and PCA extract dimensions from the (*pxp*) matrix of interattribute correlations, RMDS extracts presumably ratio distances from the (*nxn*) matrix of profile Euclidean distances, and MDA extracts dimensions from the (*nxn*) matrix of interbrand Mahalanobis distances. It is, however, the type of input rather than the order of the matrices being factored that distinguishes the solutions. Correlation coefficients ignore elevation (mean) and scatter (variance), and only information on shape (covariance) remains, whereas distance-type measures retain information on scatter and shape (Cronbach and Gleser 1953). In fact, if covariances are used instead of correlations, then it can be shown that the product spaces derived from a PCA factoring of the (*pxp*) interattribute variance-covariance matrix and from a RMDS factoring of the (*nxn*) matrix of profile Euclidean distances produce equivalent perceptual product space solutions.

The reasons for this are simple. First, factoring covariances and distance-type measures generally give similar results because both types of input retain information on scatter and shape (Nunnally 1962; Skinner 1978). Second, because the eigenstructure produced from operating on an interproduct matrix (**X'X**) and on an outer-product matrix (**XX'**) are related (see Horst 1963, Ch. 17), it can be shown that the centered scores of the brands on the first *k* principal components are identical to the brand coordinates produced by RMDS solution.[8] The relationship of MDA product space solutions to those produced by either FA, PCA or RMDS is not as straightforward, however. In general, MDA will produce solutions that differ from those of either FA, PCA or RMDS. Differences in solutions are due to the fact that MDA takes into account the within-brand interattribute covariances, and the reduced space will be oriented to variables having relatively large between-brand to within-brand variation. Thus, compared to FA, PCA, and RMDS, MDA will probably use fewer attributes; however, because the dimensions that are uncovered by RMDA can be directly linked to attributes for which there exist large brand differences, the solution may actually prove more actionable in the Shocker and Srinivasan (1974) sense.

There are several other aspects of a PCA solution that warrant some discussion.[9] First, it is well known that the PCA uncovers linear combinations of the original variables that have maximum variance (see Morrison 1976). The first principal component of the observations **X** is that linear compound $Y_1 = a_{11}X_1 + \cdots + a_{21}X_2 + a_{p1}X_p$, of the responses whose sample variance $S_{Y_1}{}^2 = \Sigma_i\Sigma_j a_{i1}a_{j1}S_{ij}$ is greatest for all coefficient vectors normalized so that $\mathbf{a}_1'\mathbf{a}_1 = 1$. In general, all else the same, the dimensions uncovered by PCA will be oriented to those variables having large variance. However, note that the sample variance of the first linear compound has terms associated with both the variances and covariances of the original variables, and if the attribute ratings tend to covary so that the off-diagonal covariance terms are large, then a rigid rotation of the original response coordinate system that parsimoniously accounts for much of the variability in the data can be accomplished regardless of the size of the variance terms. Moreover, with correlational input the role of variance becomes less compelling, since the dimensions are extracted from the standard score space.

Second, PCA based upon interattribute correlations uncovers dimensions characterized by descriptive adjectives that mean about the same thing to people; that is, PCA focuses on semantic meaning and essentially identifies groups of similar statements. Thus one can say that PCA produces dimensions characterized by attributes that are seen as similar by respondents (i.e., people agree in their ratings). As we indicated in our discussion of choice/preference models, this means that attributes that are most important in determining choice behavior could be overlooked if they were relatively independent of other attributes. Third, and finally, the importance of a dimension extracted by PCA with correlational input is determined by the *number of attributes loading on the dimension and their perceived similarity*—the more that are similar the greater the apparent importance of the dimension.

Three-Mode Models

There are two basic types of models suitable for analyzing three-way data: (1) component models or individual differences models—e.g., CANDECOMP (Carroll and Chang 1970), INDSCAL (Carroll and

[8] A longer version of this article is available from the first author upon request. In the longer version a technical appendix that documents these contentions is provided.

[9] Many of the following remarks are in conflict with the discussion appearing in Huber and Holbrook (1979).

Chang 1970), PARAFAC (Harshman 1970), and TUCKER3 (Tucker 1966); and (2) common factor models or covariance structure models (Bentler and Lee 1979; McDonald 1978; Synder 1969). The first type of model is considered determinate—or fixed in all three modes, whereas the second type has one mode, usually respondents, which is stochastic. These two types of models differ in a fundamental sense: component models are generally more "data-analytic" and exploratory, while factor analytic models are more "statistical" and confirmatory (Kroonenberg 1983). In this sense, the component models will generally be of more interest in the context of perceptual product space analysis. For this reason we will confine our remarks to this class of models.

Discussion. It is beyond the scope of this article to discuss the nuances and differences that distinguish all of the various component models for analyzing three-mode data. Interested readers should consult Harshman and Lundy (1984a) and Kroonenberg (1983) for technical details and information on computer program availability. There are, however, several aspects of three-mode component analysis that warrant some brief attention. Essentially, three-mode component models attempt to describe *each* mode of the data box in terms of a number of reduced dimensions or factors. The models assume that there is some "average" factor matrix connecting the items and the item factors. This matrix is analogous in spirit to what would have been obtained by separate component analyses for each layer of the data (e.g., brand) if these analyses had been weighted together. For example, when different brands are considered, the attribute factor matrix is differentially weighted for each brand, depending on the entries in the brand factor matrix. Assuming no brand × attribute interaction, the brand factor matrix could contain only one factor, loading on all brands, whereas with such interaction several brand factors would likely surface. The basic output from three-mode component models consists of an attribute factor matrix describing the attributes in terms of attribute factors, a brand factor matrix, and a respondent factor matrix. These three matrices are connected by means of the core matrix that indicates how the various kinds of factors are related.

Most of the three-mode component and factor analysis models assume that there is a common set of factors that generates data at all different levels of the three-way matrix.[10] Essentially, the various models only differ with respect to how the factors are weighted or combined across the different levels to account for systematic differences between successive two-way arrays. Of the more popular three-mode component models, Tucker's three-mode models are in general more flexible and less restrictive than the other formulations. However, as we discuss below, an important property of the PARAFAC/CANDECOMP models is the intrinsic axis property.

SOLUTION ISSUES

Viewed geometrically, a typical perceptual product space solution provides two basic types of information: (1) a configuration of points—i.e., brand locations—in a reduced low order dimensional space, and (2) a set of axes that span the space. Though the location of brands in the reduced space provides a compact description of the observed relationships among the brands that may clarify the patterns we are trying to understand, it does not reveal genuinely novel information. What does take us beyond the observed relationships into new inferred ones is the axes that represent the latent dimensions. In other words, it is the projections of brands onto correctly oriented axes that potentially indicates, under appropriate conditions, the relationships between the observed variables and their latent counterparts that, in turn, provide the clues to the empirical processes responsible for the observed patterns. Thus the choice and confirmation of a particular axis orientation is extremely important.

Axis Orientation

The common practice in prototypical perceptual product space analysis is to effect a rotation of the initial solution that involves rotating either the component, factor, or discriminant loading matrix, depending on whether FA, PCA, or MDA is used to extract the latent dimensions (cf. Holbrook and Huber 1979; Huber and Holbrook 1979). One reason for rotating is the search for "meaningful" interpretations. This is typically accomplished by rotating to optimize some desired characteristic of the resulting factors. The most common index is called "simple structure" (Thurstone 1947), which is frequently approximated by use of the VARIMAX method of rotation.

Discussion. A long-standing issue in traditional factor analysis is the rotation problem. The two-way factor analytic model is underidentified and, conse-

[10]The assumption of a common set of factors may not always be strictly appropriate. For example, suppose that in the typical perceptual product space setting in which a respondent × brand × attribute three-way array is analyzed, two respondents use a common dimension, but one respondent thinks of the dimension in a slightly different way from the other. In other words, suppose that the cognitive or perceptual dimensions underlying the responses may not have exactly the same quality from one respondent to another, even if they are constant within a given respondent across brands and attributes, which would imply that one respondent's pattern of factor loadings might differ from the other by more than a simple proportional reweighting. Experience has shown that the assumption of common factors but different weighting rules across the levels of the array is a useful approximation that often works well (cf. Haan 1981; Harshman and Lundy 1984a).

quently, there are an infinite number of possible solutions consistent with a given data set. Some researchers believe that this rotational indeterminacy is not a problem because the particular orientation of axes in factor-analytic solutions is not important. Different rotations of a factor-analytic solution are said to correspond to different yet equally valid perspectives on the same complex phenomenon (e.g., see Thurstone 1947, p. 332). Though this perspective may be appropriate when the purpose of the analysis is to obtain a condensed description of the data, it is less than compelling in the case of perceptual product space analysis, the purpose of which is to obtain novel information on the latent, but empirically real, processes that generate the observed relationships. Equally valid solutions will give rise to competing alternative hypotheses concerning the cognitive or perceptual processes (i.e., dimensions) that generate the data, and these hypotheses will lead to different predictions regarding consumer preference or choice.

It is important to note that since the various rotation criteria are not part of the factor model itself, additional assumptions must be imposed that may be difficult to defend on empirical grounds. For example, the likelihood of obtaining valid dimensions by rotation to simple structure depends on the belief that maximizing the simplicity criterion is appropriate for the particular situation—are the relationships between variables and underlying factors always maximally simple (see Comrey 1967, p. 143)? In addition, further ambiguities arise because of difficulties in what is viewed as "meaningful." Harshman (1970, pp. 8–14) presents an excellent discussion of the limitations of traditional factor rotation procedures.

In light of the rotational indeterminacy problem it seems natural to recommend that consumer researchers conducting perceptual product space analysis utilize extraction techniques that produce a unique set of dimensions. It is well known that PCA yields a unique solution. However, PCA will, in most application settings, prove less than totally satisfactory because of the way in which the unique solution is obtained. In PCA the axis locations are fixed by the additional requirements that the first dimension accounts for the largest percentage of the total variance and that each successive dimension accounts for as much of the remaining variance as is possible. But maximizing the variance explained by the first dimension will generally mean that the first dimension represents a combination of the processes underlying the data, rather than any one of them; as such, it often appears as an average evaluative factor. Further, as discussed by Huber and Holbrook (1979), though the evaluative dimension orders brands by the respondent's average degree of liking, it gives no information about the reasons why a brand is liked or disliked.

In the case of three-mode models, the situation is more encouraging in that the rotational indeterminacy that plagues two-way factor analysis models can be resolved. As we indicated in the previous section, the PARAFAC and CANDECOMP models possess the intrinsic axis property, which means that the solution obtained is unique up to permutation, reflection, and scalar (diagonal) transformation; note that none of these changes affects the orientation of the axes. With the PARAFAC and CANDECOMP models no rotation is permissible. Technical details on the intrinsic axis property of these models can be found in Harshman and Lundy (1984a, pp. 152–157); however, suffice it here to note that as a consequence of minimizing the error of fit, the intrinsic property ensures that the characteristics crucial for the interpretation of the dimensions are uniquely determined. In other words, the location of axes is an intrinsic characteristic of the factor solution itself. In contrast, Tucker's three-mode models allow representations in which the axes can be oriented in any position and in which all three modes can be rotated independently of each other.

Confirmation

It seems obvious to suggest that the dimensions uncovered in a perceptual product space analysis be in some sense confirmed. However, with the notable exception of the Hauser and Koppelman (1979) study, not enough attention has been given to the confirmability and stability of the axis orientation.

Discussion. The important point to note about the empirical confirmation of a perceptual product space solution is the contrast between intrinsic axis solutions and PCA (or FA) with simple structure solutions. As Harshman and Lundy (1984a, p. 166) discuss, in the case of these latter methods, the finding of consistent axis orientations across two split halves, or by other means such as the bootstrap methods, does not constitute evidence for their empirical validity. Relatively similar axis orientations necessarily occur—so long as the configurations of points in the factor spaces are similar across split halves—since these two-way methods determine axis orientation by finding directions in the configuration that maximize some simplicity or variance criterion. So long as the split-half configurations are similar, any arbitrary rotation principle based on relations of axes to points in the space would show similar results in the two split halves. Thus, this does not provide evidence for the correctness of a particular rotation criterion. In contrast, with intrinsic axis methods, a consistent configuration is not sufficient to ensure consistent axis orientations; in each split half there must be systematic stretches and contractions of the configuration as one proceeds across levels of the third mode, and these stretches must be in consistent directions in the two split halves. Thus, in this case, replication of an intrinsic axis solution validates the criterion used for orienting axes.

CONCLUSION

In this paper we have discussed several problematic issues that surface in perceptual mapping applications involving product space analysis based on compositional multi-attribute models. Among other things we have in our discussion (1) cautioned against the blind use of preliminary scale transformations, (2) investigated the relationship among alternative perceptual mapping techniques, and (3) admonished the use of aggregate perceptual product space solutions.

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