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The analysis of price competition between corporate brands

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Abstract

The methodology developed in this paper provides a means to analyze price competition between corporate brands. Corporate brands are considered to be produced and marketed by the same company. We establish price competition from an array of cross-price elasticities across time, which provides the necessary information to uncover the competitive interaction effects between corporate brands. The cross-price elasticities across time form a three-mode three-way array. The Constrained TUCKALS3-approach is developed and introduced for the analysis of the complex array of cross-elasticities. The approach takes explicitly a priori information about the competitive reaction or pattern of the marketing activities in a competitive market into account. This new methodology will enable brand management to gain further and deeper insight into the competitive interaction effects in the market. The Constrained TUCKALS3-model parameters provide the basis to investigate cannibalistic effects between corporate brands and thus helps to improve the marketing-mix, especially the price management of these brands. Furthermore, the results of the Constrained TUCKALS3-approach can serve to determine idealized market share estimates for certain a priori defined competitive conditions. The applicability and the methodological advantages of this approach will be shown by an empirical study on the price competition between two corporate brands. The reported results provide managerial useful information for the development and improvement of marketing-mix-strategies of these corporate brands. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Estimating and predicting the competitive response to changes in marketing instruments has become one of the major research topics in marketing. Much of the recent prominence of the quantitative research on marketing effectiveness is caused by the increasing availability of electronically measured buying data. In this domain, a large group of re-

searchers have focused their interest on the econometric estimation of marketing-mix cross-elasticities (e.g. Allenby, 1989; Blattberg and Wisniewski, 1989; Carpenter et al., 1988; Cooper, 1988; Krishnamurthi and Raj, 1988; Russell and Bolton, 1988). Based on aggregate sales data from a store, a chain, or a regional market, the estimated parameters of models used in this approach lead to forecasts of the overall impact of a brand's marketing policies. In addition, cross-elasticities (estimated from the model parameters) give insight into the competitive market structure (substitutability of brands) and the strengths and weaknesses of brands with respect to different kinds of marketing activities of the brands under study. Following Blattberg and Neslin (1990, p. 373), the

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key issue that still needs to be addressed is the analysis of cross-elasticities to understand competitive effects of deals. Thus, in the last decade, various tools have been developed for analyzing the complex information from the elasticity matrices.

Cooper (1988) estimates market-share elasticities from a generalized market-share response function and applies a three-mode factor analysis to the threeway array of elasticities in order to reduce the information. The result is a structural map that represents the competitive clout and receptivity of the brands for certain competitive situations. Allenby (1989) applies multidimensional scaling so that he can graphically represent the relative positions of competing brands based on the estimated cross-elasticities and their standard errors. Russell et al. (1993) develop an approach that decomposes the cross-price elasticity matrix to reveal potentially overlapping preference segments. Market-level elasticities are expressed as functions of segment weights and withinsegment market shares.

Whereas the models of Russell et al. (1993) and Allenby (1989) concentrate their analysis on the competitive pattern in the whole research period, Cooper's approach focuses on the periodical elasticity structure as well. Elasticity estimates relating to specific time periods that are derived from an asymmetric market share model can vary substantially from period to period. The dependence of the elasticity estimates on the realized actual market share during the research periods raises the question whether it is appropriate to work with average market share elasticity estimates or not. Cooper (1988) shows in his decomposition of an elasticity array that characteristic time periods or typical competitive conditions within the market can be revealed. Typical conditions may be a repeated competitive reaction or an ongoing price war between two competitors. In this respect, price and promotion campaigns at the brand or UPC level often show similar patterns within and across stores. Especially if two or more brands are strong competitors, temporary price reductions of one brand may affect the frequency and timing of competitors' price promotions. The possible pattern of price competition will surely affect the cross-price elasticity estimates of the brands across successive time periods. With respect to corporate brands, preliminary analysis of their price promotion plans has revealed that brand management often develops and applies corporate promotion plans for its brands. As a consequence, we can observe that brands of the same manufacturer are simultaneously offered at reduced prices or displayed and featured in the same weeks and same stores.

The purpose of this paper is to investigate these competitive situations. We will focus our attention on the competitive interaction effects between brands, primarily between corporate brands. Corporate brands are brands of the same product category that are produced and marketed by the same company. Though in general these corporate brands serve different consumer segments, they still may compete in certain competitive conditions, e.g. when one or both brands are sold at reduced prices. We wish to emphasize that we focus in particular on price competition and the effects of temporary price reduction on brands' competitive market positions and brands' market shares, especially with respect to corporate brands. The methodology presented in this paper will enable us to evaluate the cross-competitive effects between the corporate brands and to evaluate the promotional effectiveness of these brands. As such, the results will provide guidelines for the improvement of the promotional activities of the brands. The analysis will also provide insight into the possible cannibalistic effects between the corporate brands.

The reported research adopts the framework of decomposing elasticity matrices into their basic competitive conditions. We assume that a purely exploratory analysis as outlined by Cooper (1988) may not be sufficient to uncover the true competitive pattern in order to develop an effective marketing-mix-strategy for corporate brands. With our approach, we demonstrate the necessity of constraining the general exploratory solution to incorporate a priori information about the competition in a given market.

The integration of a priori information into the analysis enables researchers and brand managers to investigate the competitive interactions between certain brands for specific competitive conditions. These competitive conditions can be derived from game theory, e.g. independent (Nash) behavior or competitive or collusive behavior. But the competitive conditions can also be determined by the brand management. For this task, we present a constrained

three-mode component analysis model. In this respect, the integration of a priori information into the analysis allows us to investigate certain important competitive conditions and it allows us to test the competitive interaction effects of these competitive conditions. Thus, our approach provides additional information compared to the purely exploratory approach provided by Cooper (1988) about the competition and the competitive interaction effects in the analyzed market. In this way, our approach may also provide further insight into competition between brands in general and thus may provide guidelines to work on a theory of competitive interaction.

The presented research is related to—but quite different from—the new empirical industrial organization research (NEIO) (e.g. Kadivali et al., 2001). Theoretically based research within the NEIO framework has typically investigated three forms of market conduct (competitive interactions) (see e.g. Raiu and Roy, 1997): independent (Nash), leader-follower (Stackelberg) and collusive behavior. In a typical NEIO study, these different forms of competitive market conduct are estimated from the data by assuming profit-maximizing behavior of the market members and by defining a structural model that consists of demand and supply functions. Hence, the decisions of the players in the market are interdependent. In contrast to the NEIO approach, our reduced form approach is only based on a demand function (the attraction model) and implicitly accounts for the supply effects. The discussion will provide an evaluation of the NEIO approach relative to our approach.

The rest of the paper is organized as follows. Next, we briefly discuss the asymmetric market share attraction model that has been introduced by Carpenter et al. (1988) and that we use to estimate crossprice elasticities. Subsequent to this, we provide the three-mode component analysis and its constrained version to further analyze the cross-price elasticities. This approach enables us to uncover and to test certain a priori defined competitive conditions. These methods will be applied to a sample of scanner data from a consumer product market. Our research approach will clearly demonstrate how to identify and evaluate the competitive interaction effects between corporate brands for a priori defined competitive conditions. The paper concludes with a summary of the key results.

2. Investigating price competition between corporate brands

2.1. The measurement of price competitions between brands

As outlined in Section 1, the purpose of this paper is to investigate the competitive interaction effects between brands, e.g. for corporate brands. We focus particularly on price competition. The degree of price competition and the competitive cross-price effects will be established on the basis of cross-price elasticities. These cross-price elasticities are estimated for each time period (e.g. week) in order to account for the different price promotional activities of the brands across time and their impact on the structure of the cross-price elasticities.

A market response model that provides useful cross-elasticities and which is therefore useful in brand planning is the asymmetric market response model by Carpenter et al. (1988), henceforth CCHM. It reflects not only the differential effectiveness with which different brands execute their marketing strategies but also the stable cross-competitive effects. Cross-competitive effects reflect that brands differ in the degree to which they are influenced by the other brands' actions as well as the degree to which they exert influence on the other brands. The CCHM-model is

$$s_{it} = \frac{A_{it}}{\sum_{j=1}^{m} A_{jt}},\tag{1}$$

$$A_{it} = \exp(\alpha_i + \varepsilon_{it}) \prod_{k=1}^{K} \left[f_{kt}(X_{kit}) \right]^{\beta_{ki}}$$

$$\times \prod_{(k^*j^*) \in C_i} \left[f_{kt}(X_{k^*j^*t}) \right]^{\beta_{k^*ij^*}},$$

where m is the number of brands, s_{it} is the market share of brand i in period t, A_{it} is the attraction of brand i in period t, α_i is the constant brand attraction of brand i, ε_{it} is the error, X_{kit} is the level of marketing instrument k of brand i in period t, f_{kt} (·) is a double-subscripted function in which k controls whether an MCI or MNL form is used for a particular marketing instrument and t controls whether raw scores, the index of distinctiveness,

exp(z-scores), or a zeta-score transformation is used on the exploratory variables in time period t (see Nakanishi et al., 1974), β_{ki} is the parameter showing the differential effectiveness of the marketing instrument k of brand i, $\beta_{k^*ij^*}$ is the cross-effect parameter of brand j^* 's k^* th marketing instrument on brand i and C_i determines the subset of potential cross-effects for brand i.

Carpenter et al. have provided a two-step approach to estimate the model parameters. The market response parameters are tested for equality across brands to determine whether or not a brand has a different brand-specific intercept or a differential effect of a marketing instrument. Finally, the residuals from a tentatively specified model are investigated to detect a subset of potential cross-effects. As Nakanishi and Cooper (1982) showed, model 1 can be transformed to a linear specified model in logarithms with period- and brand-specific intercepts and estimated by generalized least squares.

The parameter estimates of the CCHM-model can be used to estimate cross-elasticities for each time period and each marketing instrument. The general expression is given by

$$x_{ijt}^{(k)} = \beta_{ki} \nu_{ijt}^{(k)} - \sum_{i'=1}^{m} s_{i't} \beta_{ki'} \nu_{ijt}^{(k)}$$

$$+ \sum_{(kj^*) \in C_i} \beta_{kij^*} \nu_{ijt}^{(k)}$$

$$- \sum_{(kj^*) \in C_i} s_{i't} \beta_{ki'j'} \nu_{j'jt}^{(k)},$$
(2)

$$\nu_{ijt}^{(k)} = \frac{\partial f_{kt}(X_{kit})}{\partial X_{kit}} \frac{X_{kit}}{f_{kt}(X_{kit})}, \tag{3}$$

where $x_{iji}^{(k)}$ describes the relative effect of the kth marketing instrument of brand j on the market share of brand i in period t. Thus for each marketing instrument, we can calculate a three-dimensional array of elasticities. The three-way array of elasticities specifies the direct and cross-competitive effects of the brands for the underlying time periods. The elasticity array may be seen as a three-dimensional box. Each layer of the data box corresponds to one particular time period. The ith row of one slice of the data array describes how the market share of the ith brand is influenced by the marketing-instrument

of the competitors (*receptivity*). In contrast, the *j*th column of the corresponding slice of the elasticity array indicates the competitive pressure of the *j*th brand on the competitors (*clout*); see Cooper and Nakanishi (1988, p. 183) for a graphical representation. As has been mentioned above, the slices of the data box correspond to certain time periods that represent competitive situations such as e.g. the price reduction of a particular brand or a week with regular prices. Following this interpretation, the three-dimensional elasticity array is characterized by three different modes: receptivity (mode A), clout (mode B) and time periods/competitive situations (mode C).

2.2. An exploratory approach to analyze competitive interaction effects on the basis of cross-price elasticities

A general method of uncovering the basic competitive conditions is the three-mode principal components model by Tucker (1963, 1964, 1966, 1972). The three-mode component model has been labeled by Kroonenberg and de Leeuw (1980) as Tucker3 model and it has been introduced by Cooper (1988) into the framework of market-share analyses. The Tucker3 model shows the following algebraic structure:

$$x_{ijt} = \sum_{p=1}^{P} \sum_{q=1}^{Q} \sum_{r=1}^{R} a_{ip} b_{jq} c_{tr} g_{pqr} + e_{ijt},$$
 (4)

where x_{ijt} is the (i, j, t)-element of the three-mode three-way data array, with i = 1, ..., I and j = 1, ..., J and t = 1, ..., T. In general, I and J are equal to the number of analyzed brands. a_{ip} is the (i, p)-element of the $(I \times P)$ -dimensional component matrix A, which represents the realization of the ith brand on the ith brand on the ith brands that are exposed to a similar competitive pressure in the market. i is the i is the i p-element of the i dimensional component matrix i and it describes the realization of the ith brand on the ith clout component. Corresponding to the interpretation of the component matrix i i the ith component of

matrix \boldsymbol{B} specifies those brands that exert a similar competitive pressure on the market. The loading c_{tr} of the $(T \times R)$ -dimensional component matrix \boldsymbol{C} contains the information about the relation of the tth week/competitive situation on the rth time component, which is the rth competitive condition.

The element g_{pqr} is the (p, q, r)-element of the three-dimensional "core array" G. The element provides the information of how to link the pth component of matrix A with the qth component of matrix B and the rth component of matrix C. e_{ijt} is the (i, j, t)-element of the three-dimensional error array E of dimension $(I \times J \times T)$. E exists only if the three-way elasticity array is not decomposed into all the possible components I, J and T. The formal representation of competitive interactions in the three-mode component analysis is given in Appendix A.

Focussing (for example) on price, it is important to understand that the tth week represents certain price actions of the brands under study, i.e. competitive situations. As such the tth week may represent the temporary price reductions of the market leader or it may represent the unpromoted prices of all brands in that product category. High loadings on the rth time component of matrix C determine the rth competitive condition. With respect to price activities of the brands, the real observed price promotions (e.g. temporary price reductions of the market leader or unpromoted prices of all brands in that product category) serve to determine the meaning of the rth competitive condition and help to determine the underlying competitive interaction effects between the brands in the market. However, in many markets and for many applications of this approach, the structure of the component matrix C, which represents the basic competitive conditions of the market, may not show a clear and meaningful result and a three-mode component analysis may not be able to detect a parsimonious description of the underlying competitive structure in the market.

However, the approach to decompose elasticity arrays and to investigate brand competition on the basis of the model results is purely exploratory. Furthermore, the brand management might want to test and evaluate the competitive interrelations for certain brands in prespecified competitive situations. With respect to the marketing-mix-management of corporate brands, preliminary analysis has revealed

that promotional activities of corporate brands can be highly coordinated across time. The pattern of the promotional diary reveals that corporate brands are promoted in the same weeks of identical stores or chains. As such, brand management should be highly interested in evaluating possible cannibalistic effects between its brands. Therefore, it is necessary to detect the competitive interactions between brands in different situations, e.g. simultaneous price promotions, alternating price promotions or unpromoted prices in order to optimize the marketing-mix. The interaction effects must be quantified and possible cannibalistic effects have to be determined for an effective and powerful marketing-mix-management. The relevance of this discussion is encouraged by the fact that most consumer goods markets have an oligopolistic structure. Few companies distribute several brands in the same product category.

The analysis of prespecified competitive situations/conditions cannot be realized within the general exploratory three-mode component analysis. But a possible solution can be seen in constraining the three-mode component model. Therefore, the general approach to decompose elasticity arrays has to be modified so that brand management can investigate certain prespecified competitive situations/conditions.

2.3. The analysis of a priori defined competitive conditions

We introduce a constrained three-mode model to force the parameter estimates of matrix C to follow a prespecified structure. To illustrate this, assume that brand A and brand B are corporate brands. Brand management may employ for several periods identical promotional activities for these two brands. e.g. offering trade promotions for both brands in the same time period so that both brands may be pricepromoted in the stores in identical weeks. Obviously, these simultaneous trade promotions do not need to be optimal with respect to sales or market shares. As such, brand management needs to evaluate the cannibalistic effects between brand A and brand B in order to determine the effectiveness of simultaneous price promotions. Insight into this problem could be gained from the competitive conditions, which are represented in component matrix C of the Tucker3 model. If this matrix were representing the brand competition between brand A and brand B (e.g. A and B promoted at the same time), we would be able to determine the competitive interaction effects between the two brands. Thus, the component matrix C should have components that represent the simultaneous price promotions of A and B, temporary price reductions of A (B) while B (A) is sold at regular prices or the case where both brands are sold at regular prices.

2.3.1. Estimation methods for the representation of a priori defined competitive conditions

A solution to the problem of investigating the competitive interaction effects for certain brands will be given by a Constrained TUCKALS3-algorithm. It enables us to estimate the component loadings with the restrictions provided by the design or target matrix. But first we will relate this method to alternative methods, which may also be used for a "constrained" parameter estimation. The information for the constraints are provided by the promotional diary. For the analysis of price competition, we will determine the constraints on the basis of the observed temporary price reductions in the market. Thus, we are able to determine different competitive conditions that form the basis of the so-called design or target matrix.

Basically, three different approaches can be distinguished for the constrained parameter estimation within the Tucker3 framework: Procrustean transformation after parameter convergence, External TUCKALS3-analysis and Constrained TUCKALS3analysis. Procrustean transformation (see Harman. 1976) is based on the estimated results of the unconstrained TUCKALS3-solution and—in our case tries to maximize the congruence of a component matrix with the given target matrix. One disadvantage of the Procrustean approach, however, derives from the two-step procedure. Another major disadvantage of the Procrustean approach is that the "constrained" solution may not be found so that the existence of the postulated structure has to be rejected. Furthermore, each element of the design or target matrix must be explicitly defined.

The external TUCKALS3-analysis is based on one or more orthonormal component matrices that are fixed a priori and only the parameters of the remaining component matrices and the core array are estimated (van der Kloot and Kroonenberg, 1985). The approach may be seen as an alternative to the Procrustean approach if the elements of the external given matrix are a priori known (in the Procrustean approach the given structure serves as the target matrix). Unlike to the Constrained TUCKALS3-approach (to be presented below), both methods (the Procrustean transformation and the external TUCK-ALS3-approach) need a fully specified design or target matrix. Consequently, all the weeks that contribute to one component of the design matrix have to be assumed as equally important.

The Constrained TUCKALS3-method is based on a reformulation and modification of the TUCK-ALS3-estimation algorithm, which allows the integration of linear equality constraints on a subset of parameters. Now the parameter estimation of one or more component matrices follows the pattern of the given design or target matrix. The overall amount of variance will be reduced because some elements of a component matrix are fixed to zero. On the other hand, only the zero elements are fixed and the other parameters are freely estimated. Thus, the Constrained TUCKALS3-approach allows differentially weighted time periods. Moreover, the parameter estimates are derived from a simultaneous estimation procedure. Constrained TUCKALS3 optimizes only one target criterion.

2.3.2. The constrained TUCKALS3-approach

In order to constrain certain parameters in the Tucker3-model, we adopt the algorithm of Weesie and van Houwelingen (1983) and complement it by the possibility of linear equality constraints on a subset of parameters. The integration of linear equality constraints is based on the LSE algorithm outlined by Lawson and Hanson (1970). The linear equality constraints are held in the binary design matrix D. Zeros indicate the elements that are considered to be zero, the ones indicate the parameters that are estimated by minimizing the Constrained TUCKALS3-loss function. The constrained parameter estimation is shown for the component matrix C. The procedure of the general algorithm is outlined first for the parameter estimation of component matrix A.

The parameter estimates of the Constrained TUCKALS3-analysis can be derived from a modification of the TUCKALS3-loss function

T3(
$$A$$
, B , C , G ₁,..., G _R)

$$= \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{t=1}^{T} \left(x_{ijt} - \sum_{p=1}^{P} \sum_{q=1}^{Q} \sum_{r=1}^{R} a_{ip} b_{jq} c_{tr} g_{pqr} \right)^{2}$$

$$= \|X - AG(C' \otimes B')\|^2 \tag{5}$$

with X as $(I \times TJ)$ -dimensional array of the crosselasticities, A, B and C are the $(I \times P)$ -, $(J \times Q)$ and $(T \times R)$ -dimensional component matrices, G is the $(P \times RQ)$ -dimensional core array and \otimes indicates the Kronecker-product. An estimate of the component matrix A is achieved within an alternating least square algorithm. If the component matrices B and C and the core array G are considered to be fixed (based on their previous estimates), an estimate of the component matrix A can be derived by

CT3(
$$A$$
) = $||X - AG(C' \otimes B')||^2 = ||X - AF||^2$
(6)

with X as a $(I \times TJ)$ -dimensional matrix, G as the $(P \times RQ)$ -dimensional core array, F as the $(P \times TJ)$ -dimensional matrix $F = G(C' \otimes B')$ and $A' = (FF')^{-1}FX'$. Similar transformations have to be performed for the component matrix B with A, C and G fixed and for the component matrix C with A, B and G fixed. An estimate of the three-dimensional core array may be obtained by the following procedure

$$CT3(G) = \|Vec(X) - (A \otimes C \otimes B)Vec(G)\|^{2}$$
$$= \|Vec(X) - FVec(G)\|^{2}$$
(7)

with X as a $(TI \times J)$ -dimensional matrix, G as the $(P \times RQ)$ -dimensional core array, F as the $(IJT \times PQR)$ -dimensional matrix $F = A \otimes C \otimes B$. Vec (\cdot) indicates the Vec-operator that creates a column vector by appending the columns of a matrix to each other. If a subset of the parameter estimates of the component matrix C are fixed to a linear equality constraint, the parameter estimation has to be done separately for each row of the constrained component matrix (for details see Appendix B). In general, the outlined procedure will guarantee a global, unique solution, irrespective of starting values.

After the component matrices of the Constrained TUCKALS3-model and the core array have been estimated, the idealized elasticities or competitive maps may be estimated by Eqs. (3)–(5). An interesting additional option can be seen in the estimation of idealized or expected market shares for the a priori defined competitive conditions. These shares can be estimated by weighting the real market shares with the estimated component loadings of matrix C from the Constrained TUCKALS3-analysis and normalizing the estimates by the weighted sum of shares

$$s_{ir} = \sum_{t=1}^{T} s_{it} c_{tr} / \sum_{j=1}^{m} \sum_{t=1}^{T} s_{jt} c_{tr},$$
 (8)

where s_{ir} is the expected market share of brand i in the rth competitive condition. This rth competitive condition refers to the rth component of matrix C of the constrained solution.

3. The empirical application

To illustrate our approach, we analyze estimates of cross-price elasticities of a German consumer goods market. We apply the CCHM-model to estimate direct and cross-competitive effects. The resulting parameters serve for the estimation of a three-way array of price elasticities. We then use the knowledge about the promotional activities in the market to estimate the competitive effects for specific competitive situations. In particular, we investigate the competitive interaction effects between two corporate brands. On the basis of these results, idealized elasticities and idealized market shares for a given and well-defined promotional pattern are derived. The necessary analyzing tools have been implemented in the statistical programming language GAUSS.

3.1. Data description

Our example is based on scanner panel data from an instrumental test market provided by the GfK, Nuremberg. The data are measured at the store-level and cover a period of 104 weeks. The product category is body care. Within this product category, 65 different brands are offered, some only for a few weeks. The market is dominated by nine brands, which account for more than 82% of the total market. The nine brands are distributed in unipacks and multipacks. The remaining 56 brands are characterized by a very high degree of heterogeneity of prices, physical characteristics, distribution shares and the use of marketing instruments. For these reasons, we concentrate on the nine major brands. The brand names are disguised so that they are labeled as B1, B2, etc. In addition, the analysis of market shares across stores emphasizes the analysis of store specific data, as the market shares and distribution across stores is very heterogeneous.

The marketing instruments in the market under study are price, display and feature. Our analysis about the competition between the corporate brands will only focus on price competition. Table 1 gives a survey of the observed market shares and marketing instruments for the total period as well as for the calibration sample (the first 96 weeks) and the validation sample (the last 8 weeks).² Temporary price reductions and regular prices have been identified by a kind of baseline estimation procedure applied to time series price data. All prices have been divided by the lowest price across all brands and across all weeks before calculating the mean prices for each brand in each sample (in order to disguise the name of the product category). After an exploratory analysis of the cross-price elasticities using the TUCK-ALS3-approach, we will investigate the competitive interaction effects between the corporate brands B3 and B4 using the Constrained TUCKALS3-approach. B3 and B4 are produced and marketed by a major international company in the consumer goods market. Interestingly, not only the price promotions but

also display- and feature-activities of these two brands are highly correlated across time and within stores. In fact, in more than 50% of cases, B3 or B4 is on sale the other corporate brand is also on sale in the same store. Obviously, this price pattern is caused by the manufacturer's marketing-mix-strategy, e.g. its trade promotion program. However, it is not evident whether this strategy is optimal with respect to aggregate shares or sales across the two corporate brands. Thus, we will use the promotional diary to construct a design or target matrix that will enable us to measure the competitive interaction effects between the two brands for certain a priori defined competitive conditions.

3.2. The measurement of cross-price elasticities

The measurement of cross-price elasticities and thus of brand competition between the nine brands is based on the parameter estimates of the asymmetric market share model by Carpenter et al. (1988). We use the MNL-model specification for the CCHMmodel. Prices have been transformed to z-scores and the index-of-distinctiveness (see Cooper and Nakanishi, 1988, pp. 69–78) is used for the qualitative promotional instruments display and feature. This procedure ensures that the marketing instruments reflect the degree of competition in each period, i.e. the index of distinctiveness as well as z-scores account for the number of brands that (for example) were featuring or price-promoted in a particular week. Furthermore, the variable transformations significantly reduce collinearity in the data. Following the estimation procedure outlined by Carpenter et al. differential effects were found for the brand intercepts, prices, display- and feature-actions. Based on the estimation results of the differential analysis, a cross-correlation analysis were conducted and 13 potential cross-effects were identified (four prices, five display and four feature). Thus, the market reflects only moderate asymmetric competition. We use generalized least squares to estimate the model parameters and we adjust the estimation for heteroskedastic and autocorrelated errors.

The final asymmetric market share model explains 97.5% of the variance with an *F*-value of 193.0 with 144 estimated parameters and 686 de-

¹ Marketing-mix interactions were not explicitly included into the model in order to reduce parameterization problems. The parameters for price are estimated while we control for the effects of display and feature at the same time.

² The relatively small number of validation weeks is motivated by the subsequent constrained three mode analysis. We wanted to keep as many specific competitive situations as possible in the model. Because of that, we estimated the parameters on the basis of the first 96 weeks in which the number of brands in the market was complete. During the last 8 weeks of our data sample, two brands disappeared as competitors, they may have been either out of stock or out of shelf.

Table 1
Market shares, prices, display and feature promotions in the data

Brand	Market shares			Prices			Regular prices		
	Total sample 104 weeks	Calibration sample 96 weeks	Validation sample 8 weeks	Total sample 104 weeks	Calibration sample 96 weeks	Validation sample 8 weeks	Total sample 104 weeks	Calibration sample 96 weeks	Validation sample 8 weeks
B1	5.10	5.53	0.00	1.86	1.86	1.96	1.88	1.87	1.96
B2	10.01	9.77	12.86	1.64	1.64	1.67	1.68	1.68	1.67
В3	4.87	4.79	5.78	1.53	1.51	1.71	1.56	1.55	1.73
B4	10.21	10.10	11.61	1.59	1.58	1.71	1.65	1.64	1.73
B5	12.10	12.35	9.10	2.30	2.28	2.46	2.35	2.34	2.46
B6	3.26	3.53	0.00	1.69	1.69	1.96	1.71	1.69	1.96
B7	24.98	24.72	28.18	1.70	1.68	1.88	1.78	1.77	1.91
B8	13.26	13.60	9.09	1.45	1.45	1.41	1.47	1.48	1.41
B9	16.20	15.60	23.38	1.77	1.77	1.72	1.80	1.81	1.76

Brand	# Temp. price reductions			# Display w	# Display weeks			# Feature weeks		
	Total sample 104 weeks	Calibration sample 96 weeks	Validation sample 8 weeks	Total sample 104 weeks	Calibration sample 96 weeks	Validation sample 8 weeks	Total sample 104 weeks	Calibration sample 96 weeks	Validation sample 8 weeks	
B1	7	7	0	19	19	0	1	1	0	
B2	7	7	0	12	12	0	2	2	0	
B3	14	13	1	25	23	2	5	5	0	
B4	22	21	1	22	19	3	13	13	0	
B5	7	7	0	25	25	0	4	4	0	
B6	3	3	0	1	1	0	1	1	0	
B7	28	27	1	60	56	4	13	12	1	
B8	15	15	0	41	41	0	14	14	0	
B9	13	12	1	49	42	7	7	6	1	

grees of freedom. All potential cross-price effects are significant at p < 0.10 and have been included into the estimation of the cross-price elasticities across the 96 weeks. The average cross-price elasticities are

shown in Table 2. There are seven negative crossprice elasticities. Negative cross-price elasticities are in our product category "body care" not unusual. Loyal consumers may be attracted by price promo-

Table 2
The average cross-price elasticities

	B1	B2	В3	B4	B5	В6	В7	В8	В9
B1	-2.71	0.20	-0.08	1.07	1.24	-0.27	0.20	0.00	0.33
B2	0.19	-3.66	0.32	0.43	0.58	0.85	0.38	0.46	0.42
В3	0.17	0.82	-4.93	0.77	-0.28	0.12	0.71	1.05	1.60
B4	0.33	0.69	0.55	-3.99	0.61	0.05	0.53	0.63	0.61
B5	0.70	0.14	-0.44	0.06	-1.80	0.97	0.20	-0.48	0.62
B6	0.25	0.34	0.19	0.31	0.81	-2.93	0.31	0.29	0.39
B7	0.05	0.23	0.07	0.17	0.61	-0.33	-1.22	0.17	0.24
B8	0.04	0.51	0.45	0.43	0.02	-0.15	0.38	-2.05	0.37
B9	0.26	0.35	0.12	0.31	1.04	-0.19	0.30	0.24	-2.45

tion for certain brands to the total product category. They subsequently may purchase other brands than those that were price promoted. In addition, we emphasize that most of the negative cross-price elasticities are close to zero.

The calibration results are also used for validation in the holdout sample of 8 weeks. The cross-correlation coefficient is 0.922 and the model parameters explain 84.9% of the variance in the holdout sample. Two brands were not in the market in our validation sample so we find a somewhat lower explained variance than that in our calibration sample. However, if we take the out-of-market situation of brands B1 and B6 into account, we get a cross-correlation coefficient of 0.951 in our validation sample and the explained variance is 89.45%.

3.3. The exploratory analysis of brand price competition

In the first step of the analysis, the three-way array of price elasticities is decomposed in a strictly exploratory way into basic competitive components. In contrast to Cooper (1988), we use the TUCK-ALS3-algorithm to get optimal parameter estimates according to the least squares criterion. Before the model can be applied to the data, the dimensionality of the solution must be determined. According to Tucker (1966) and in agreement with Kiers (1991), separate singular value decompositions (SVD) have to be performed on the three cross-product matrices of two-way supermatrices. A supermatrix is a reorganization of a three-way array to a two-dimensional data matrix. Each mode is the column mode of a two-dimensional supermatrix with the other modes as the corresponding rows in one of the three supermatrices (for technical details, see e.g. Kiers, 1991). The SVD of the three cross-product matrices indicate that a $(4 \times 4 \times 7)$ -dimensional solution should be appropriate to cover a large amount of variance in the data.³ Following ten Berge et al. (1987), it is possible to determine upper limits for the maximum explained variance in each mode for the selected dimensionality. Four components in mode A (receptivity) would explain 77.48% of the variance, four components in mode B (clout) would explain 78.00% variance and seven components in mode C (weeks/competitive situations) would account for 96.60% of the variance if separate analyses are done. The final TUCKALS3-solution explains 73.66% of the variance and it confirms a good fit of the model to the data

We will not go into detail about the interpretation of the exploratory analysis. Component matrix A represents the nine brands on the four receptivity components. Accordingly, the matrix **B** represents the nine brands on the four clout components. As these two component matrices already collect some information about the competitive structure in the market, deeper insights into the basic competitive interaction effects are obtained from the components of the time mode, which represents the characteristic competitive conditions. The original (96×7) dimensional component matrix C will not be shown. An interpretation of this matrix is hampered by its high complexity. To solve this problem, we have rotated the component matrix C with the VARI-MAX procedure, which coincides in case of orthonormal component matrices with the Harris and Kaiser (1964) ortho-oblique rotation that has been proved to produce component matrices of low complexity (e.g. Hakstian, 1971; Kiers and ten Berge, 1994).

In order to get a proper interpretation of the component structure, we have examined the correlations of the VARIMAX-rotated component matrix C with the prices of the nine brands. A significant negative correlation may indicate that the component represents price actions of the corresponding brand. The first, second and sixth component cannot be attributed to a meaningful interpretation because many weeks have (unsystematically) high loadings that correspond to different promotional activities in the market. The significant corresponding correlations are fairly low and range between 0.23 and 0.48. Nevertheless, we interpret the components such that the third component represents price promotions of the corporate brands B3 and B4, the fourth component represents opposite price competition between brands B2 and B6, component five corresponds to

³ Interested readers can obtain a list of all singular values from the three singular values decompositions of the two-way supermatrices

regular prices of B4 and finally the seventh component may possibly represent price promotions of B5. The results of this exploratory analysis of the three-way array of price elasticities are on the whole not convincing. The underlying competitive structure is to complex to uncover it by a parsimonious description of the exploratory approach. Consequently, the interpretation of the competitive maps and the idealized elasticities of the seven competitive conditions of the exploratory analysis remains unclear.

3.4. The analysis of price competition between brands B3 and B4

The analysis of the competitive interaction effects between the corporate brands B3 and B4 is based on a Constrained TUCKSALS3-analysis as outlined in Section 2.3.2. The component matrix, which represents certain competitive conditions, is constrained such that it follows the structure of a prespecified design or target matrix. This design or target matrix must provide the necessary information to detect the competitive interactions between B3 and B4. The information to construct the target matrix is obtained from the price promotion diary of all brands. From the brand management perspective, it is important to understand how B3 and B4 compete in alternative competitive conditions. As previously discussed, in more than 50% of B3's price promotions B4 is also on sale and vice versa. Obviously, brand management must evaluate the effectiveness of its pricing policy. Cannibalistic effects can be derived on the basis of the idealized cross-price elasticities for certain competitive conditions. We can also determine the expected shares or sales of these brands for the defined competitive conditions. Thus, in order to effectively evaluate the pricing strategies of B3 and B4, the brand management should compare the cannibalistic effects between B3 and B4 and the corresponding market shares when both brands are on sale with those cannibalistic effects when only B3 or B4 are on sale (a managerial option for the brand management) as well as with those cannibalistic effects when both brands are offered at regular prices in the stores (also a managerial option for the brand management).

Thus, the important competitive conditions to detect the price competition between B3 and B4 are

held in the target components "B3 on sale while B4 is offered at regular prices", "B4 on sale while B3 is offered at regular prices" and "B3 and B4 are simultaneously on sale". In order to account and control for competitive effects of the competing brands (B1. B2, B5, B6, B7, B8 and B9), we will specify an additional target component that represents price promotions of these brands. This component and its corresponding solution will not be analyzed. Moreover, we also model the unpromoted price condition for all brands. The regular price condition will serve to estimate baseline shares in which no brand is on sale. In this way, it will enable us to evaluate the net effect of certain price promotions. Thus, our design or target matrix has five competitive conditions that provide the necessary framework to determine the basic competitive interaction effects between the corporate brands B3 and B4.

The analysis of the Constrained TUCKALS3model is now based on the design or target matrix with the five competitive components: "B3 on sale while B4 is offered at regular prices", "B4 on sale while B3 is offered at regular prices", "B3 and B4 are simultaneously on sale", "Other brands than B3 or B4 are on sale" and "All brands are offered at regular prices". The estimation was based on the following information about the competitive situations. Brand B3 on sale while brand B4 is offered at regular prices occurred in 4.2% of cases, brand B4 on sale while brand B3 is offered at regular prices occurred in 12.5.4% of cases, brands B3 and B4 simultaneously on sale occurred in 9.4% of all cases and the competitive condition in which all brands were offered at regular prices occurred in 25.0% of cases. This target matrix is binary. Values of 1 indicate these parameters that have to be estimated by the Constrained TUCKALS3 approach while the values of 0 indicate that parameters are held fixed throughout the estimation procedure. The Constrained TUCKALS3-analysis explains 69.73% of the variance. Compared to the unconstrained solution this represents a loss of 3.89% in fit, but now the competitive conditions have a clear interpretable pattern. If we also take into account that the unconstrained solution has two additional components in mode C, this loss in explained variance appears negligible. If the exploratory analysis were done with five components in mode C, the loss in explained

Table 3
Idealized cross-price elasticities of B3 and B4 for the competitive
condition: "B3 on sale while B4 is offered at regular prices"

Impact	Clout of		Impact	Receptivi	ity of
on	В3	B4	of	В3	B4
B1	0.03	1.27	B1	0.26	0.85
B2	0.47	0.23	B2	0.86	0.51
B3	-3.66	0.53	B3	-3.66	0.68
B4	0.68	-4.20	B4	0.53	-4.20
B5	-0.81	0.20	B5	-0.83	0.86
B6	0.29	-0.06	B6	0.41	-0.23
B7	0.13	0.00	B7	0.42	0.33
B8	0.23	0.41	B8	0.64	0.77
B9	0.65	0.23	B9	1.39	0.45

variance would amount to 3.55%. We will not discuss the component matrices separately, but we want to mention that the component loadings of matrix C vary substantially within each competitive condition. Different weeks (price actions of one brand) are not equally important or powerful. The strength of one particular week also depends on the uniqueness of a competitive action.

According to the results competitive maps and idealized elasticities can be estimated for the five a priori defined competitive conditions. We will focus our attention on the idealized elasticities since the competitive maps are bipolar representations of the idealized elasticities. Table 3 shows the idealized cross-price elasticities of B3 and B4 for the competitive condition "B3 on sale while B4 is offered at regular prices". Table 3 presents the main effects as follows: The left half represents the competitive strength or clout of B3 and B4 on the competing brands. Thus, the left part of this table provides the information how the temporary reduced prices of B3 and the regular prices of B4 affect the shares of all brands. The right part of Table 3 presents the receptivity or vulnerability of B3 and B4 with respect to the prices of the nine brands in the market when B3 is on sale while B4 is offered at regular prices.

We start our discussion on the competitive strength of B3 and B4. When B3 is on sale, it can expect a direct price elasticity of 3.66. This value is lower than the average direct price elasticity (see Table 2) indicating that further price reductions for B3 will provide a relatively smaller increase in market share. If B3 is on sale, it primarily affects the market shares

of B2, B4 and B9 while B5, the highest priced brand in the market, is unaffected by price promotions of B3. However, it is important to note that B3 also gains market share from B4, the corporate brand. With respect to the competitive strength of B4, we can ascertain that B4 primarily affects the shares of B1 while B3 is on sale. Obviously, the share of B4 is very sensitive to its price. Thus, B4 could gain a relatively large proportion in share if it reduced its price. Focussing on the receptivity of B3 and B4, Table 3 reveals that the share of B3 is primarily affected by the prices of B2, B4, B6, B7, B8 and B9. Thus, most brands in the market can easily reduce the market share of B3 by lowering their own price. This indicates that the competitive strength of B3 is primarily caused by its own temporary price reduction and not by non-price features such as brand strength. However, the receptivity of B4 also reveals that the market position of B4 is not very strong in this competitive condition. The brands B1, B2, B3, B5, B8 and B9 may easily affect the market share of B4 by reducing their prices. In addition, the market share of B4 is also very sensitive to its own price. On the whole, this competitive condition where B3 is on sale while B4 is offered at regular prices does not seem to be a favorable pricing strategy with respect to the competitive strength of the two brands. The analysis of the idealized market shares later in the discussion will provide additional insight.

We will now focus our attention on the competitive condition where B4 is on sale while B3 is offered at regular prices. The corresponding results are presented in Table 4, which has the same struc-

Table 4 Idealized cross-price elasticities of B3 and B4 for the competitive condition: "B4 on sale while B3 is offered at regular prices"

Impact	Clout of		Impact	Receptivity of		
on	В3	B4	of	В3	B4	
B1	-0.25	1.41	B1	-0.12	0.75	
B2	0.29	0.55	B2	0.72	0.66	
B3	-4.47	1.24	B3	-4.47	0.45	
B4	0.45	-2.43	B4	1.24	-2.43	
B5	-0.56	-0.46	B5	-0.42	-0.13	
B6	-0.03	0.61	B6	0.06	0.21	
B7	0.04	0.19	B7	0.50	0.12	
B8	0.42	0.00	B8	0.92	0.15	
B9	0.64	0.29	B9	1.58	0.23	

ture as Table 3. Table 4 reveals that price promotions of B4 affect primarily the shares of brands B1 and B3 but also those of B2 and B6. Again, B5 is unaffected by the price promotion. Thus, if B4 is on sale, this brand will gain much of its share from B1 and the corporate brand B3. B4's own price elasticity is fairly low, indicating that additional price reductions will provide a relatively small increment in market share. With respect to B3, we can ascertain that the price of B3 primarily affects the share of B4, B8 and B9. B3's own price elasticity coefficient indicates that its market share is sensitive to its price. Nevertheless, the relative effect of B4 on B3 is almost three times as high as that of B3 on B4. If B4 is on sale while B3 is offered at regular prices, the market share of B4 is fairly protected. Only B1 and B2 can affect the market share of B4. This result indicates that temporary price reductions of B4 provide a favorable competitive situation for B4. However, the relative competitive position of B3 is affected by B2, B4, B7, B8 and B9. Thus, the price promotion of B4 affects the market position of B3 such that B3's market share is very sensitive with respect to the prices of the other market members. Especially if B9 were reducing its price, it could gain share from B3. On the whole, this competitive condition appears to be more favorable with respect to both corporate brands. In particular, the market position of B4 appears to be stronger if B4 is on sale than the market position of B3 when B3 is on sale. However, B4's temporary reduced prices affect to a large extent the market share of B3.

Table 5 presents the idealized cross-price elasticities of B3 and B4 for the competitive condition when both brands are simultaneously on sale. The clout coefficients reveal that B3's prices affect primarily the market share of B4 but also that of B9. B5 is again unaffected by the price promotion of B3. The prices of B4 on the other hand affect the share of B1 and strongly affect that of B3. Thus, we can observe strong cannibalistic effects between the two brands when they are simultaneously on sale. Both brands do not present strong cross-competitive effects on the other brands, but their own market positions are only moderately affected by the other brands. This result can be obtained from the receptivity coefficients. The market share of B3 is largely affected by the price promotion of the corporate brand B4 and it

Table 5
Idealized cross-price elasticities of B3 and B4 for the competitive condition: "B3 and B4 are simultaneously on sale"

Impact	Clout of		Impact	Receptiv	ity of
on	В3	B4	of	В3	B4
B1	-0.24	0.88	B1	-0.35	0.32
B2	0.27	0.26	B2	0.42	0.32
B3	-2.81	1.30	B3	-2.81	0.80
B4	0.80	-2.13	B4	1.30	-2.13
B5	-0.45	0.00	B5	0.00	0.63
B6	0.04	0.17	B6	-0.25	-0.34
B7	0.05	0.05	B7	0.28	0.10
B8	0.17	0.04	B8	0.56	0.29
B9	0.39	0.05	B9	0.86	0.01

is also sensitive with respect to the prices of B2, B8 and B9. Compared to the competitive market condition in which only B3 is on sale, these values are now considerably lower. This indicates that the simultaneous price reductions of B3 and B4 are able to create a market condition in which the competing brands lose much of their marketing effectiveness (here price-effectiveness). With respect to the receptivity of B4, we can ascertain the same effect. Now only the prices of B5 affect the share of B4. Thus, while B4's market share is very sensitive to the prices of B1 and B2 when it is not simultaneously on sale with B3 (first competitive condition), now the relative effects of B1's and B2's prices have more than halved. On the whole, this competitive condition appears to be favorable with respect to the two corporate brands. Though their temporary reduced prices affect the shares of the corporate brand to large extent, their market shares are protected regarding the prices of the other competing brands.

Finally, we will focus our attention on the competitive strengths and weaknesses of B3 and B4 when all brands are offered at regular prices. The necessary information is provided by Table 6, which has the same structure as the tables discussed above. In this competitive condition, the prices of B3 primarily affect the shares of B4, B8 and B9 while B1's and B5's market shares are unaffected by the prices of B3. The direct elasticity of B3 reveals that B3 can gain a large amount of share if it were reduced its price below the regular price. This is also true for B4. At regular prices, B4 affects the market position of B1 in particular but also that of B2 and B3.

Table 6 Idealized cross-price elasticities of B3 and B4 for the competitive condition: "all brands are offered at regular prices"

Impact	Clout of		Impact	Receptivi	ty of
on	В3	B4	of	В3	B4
B1	-0.25	1.43	B1	0.14	0.95
B2	0.32	0.40	B2	0.99	0.76
В3	-4.97	0.75	B3	-4.97	0.50
B4	0.50	-4.09	B4	0.75	-4.09
B5	-0.64	0.02	B5	-0.65	0.52
B6	0.00	0.12	B6	0.25	-0.08
B7	0.06	0.05	B7	0.60	0.31
B8	0.46	0.33	B8	1.05	0.65
В9	0.72	0.27	B9	1.87	0.50

Compared to the competitive market condition where B3 and B4 are simultaneously on sale, the relative effects of B3 on B4 and of B4 on B3 are only half as strong. With respect to the receptivity of B3 and B4, we can ascertain that the market share of B3 is largely affected by the prices of B2, B4, B7, B8 and B9. B9 especially has a dominant competitive impact on the share of B3. The share of B4 on the other hand is affected by the prices of B1, B2, B3, B5, B8 and B9. The receptivity coefficients for B3 and B4 reveal that the price competition at the regular price level is strong. Almost all brands can gain market share from B3 and B4 by reducing their prices. On the whole, the competitive condition of regular prices for all brands in the market shows strong competitive effects on B3 and B4. Thus, the competitive position of these two brands is very sensitive to the prices of the competing brands.

After having established the competitive position of the two corporate brands, we will now focus our attention on the expected or idealized market shares for the four competitive conditions discussed above. The market share estimates are provided in Table 7. The idealized market share estimates of the competitive condition in which all brands are offered at regular prices serves as a baseline. The analysis of the first competitive condition where B3 is on sale while B4 is offered at regular prices reveals that B3 can expect a market share of 20.57%, which is almost five times higher than the share B3 can expect when no brand is on sale. With respect to B4. we observe that B4 would lose approximately one third of its share compared to the regular price condition. With the exception of B3 and B7, all brands lose market shares if B3 is on sale compared to the condition where all brands are offered at regular prices. The two corporate brands would realize an aggregate market share of 25.83%, which is almost 6% less than the aggregate market share if B4 is on sale while B3 is offered at regular prices. In the last mentioned competitive condition, B4 can increase its market share, compared to the condition of regular prices, approximately by a factor of three. However, even B3 profits from this competitive condition because its expected market share increases by approximately 1%. All other competitors will lose share compared to the condition of regular

Table 7 Idealized market-share estimates for the a priori defined competitive conditions

	Competitive conditions			
	B3 on sale, B4 offered at regular prices	B4 on sale, B3 offered at regular prices	B3 and B4 are simultaneously on sale	All brands are offered at regular prices
B1	5.64	5.13	2.31	8.09
B2	8.46	9.75	7.05	12.24
В3	20.57	5.35	14.19	4.30
B4	5.26	26.33	23.60	8.48
B5	8.16	10.84	19.21	11.41
B6	3.59	2.84	1.40	3.53
В7	29.34	17.14	15.59	24.70
B8	6.70	6.89	7.95	10.13
В9	12.29	15.73	8.70	17.12
B3 + B4	25.83	31.68	37.79	12.78

Table 8
Bootstrap idealized cross-price elasticities and their standard deviations of B3 and B4 for the competitive condition: "B3 on sale while B4 is offered at regular prices"

Impact	Clout of				Impact	Receptivity	y of		
on	B3		B4		of	B3		B4	
	Mean	SD	Mean	SD		Mean	SD	Mean	SD
B1	0.02	0.17	1.26	0.14	B1	0.28	0.09	0.86	0.14
B2	0.46	0.11	0.23	0.05	B2	0.87	0.13	0.52	0.08
B3	-3.63	0.85	0.52	0.15	В3	-3.63	0.85	0.66	0.15
B4	0.66	0.15	-4.18	0.66	B4	0.52	0.15	-4.18	0.66
B5	-0.80	0.12	0.19	0.11	B5	-0.90	0.13	0.82	0.27
B6	0.27	0.19	-0.06	0.10	B6	0.44	0.10	-0.20	0.11
B7	0.13	0.07	0.00	0.05	B7	0.42	0.14	0.32	0.09
B8	0.23	0.15	0.40	0.14	B8	0.63	0.28	0.75	0.23
B9	0.64	0.08	0.23	0.08	B9	1.40	0.37	0.46	0.18

Table 9
Bootstrap idealized cross-price elasticities and their standard deviations of B3 and B4 for the competitive condition: "B4 on sale while B3 is offered at regular prices"

Impact	Clout of				Impact	Receptivity	y of		
on	B3		B4		of	B3		B4	
	Mean	SD	Mean	SD		Mean	SD	Mean	SD
B1	-0.25	0.09	1.41	0.11	B1	-0.12	0.08	0.74	0.06
B2	0.29	0.09	0.56	0.13	B2	0.72	0.09	0.65	0.07
В3	-4.47	0.11	1.25	0.17	В3	-4.47	0.11	0.45	0.16
B4	0.45	0.16	-2.43	0.22	B4	1.25	0.17	-2.43	0.22
B5	-0.56	0.12	-0.45	0.11	B5	-0.44	0.14	-0.14	0.10
B6	-0.04	0.12	0.60	0.15	B6	0.06	0.05	0.21	0.06
B7	0.04	0.05	0.19	0.06	В7	0.51	0.06	0.12	0.04
B8	0.40	0.06	0.02	0.11	B8	0.92	0.09	0.16	0.08
B9	0.64	0.07	0.30	0.09	B9	1.58	0.08	0.24	0.07

Table 10 Bootstrap idealized cross-price elasticities and their standard deviations of B3 and B4 for the competitive condition: "B3 and B4 are simultaneously on sale"

Impact	Clout of				Impact	Receptivity of			
on	B3		B4		of	В3		B4	
	Mean	SD	Mean	SD		Mean	SD	Mean	SD
B1	-0.23	0.12	0.90	0.18	B1	-0.34	0.26	0.33	0.19
B2	0.27	0.04	0.28	0.10	B2	0.43	0.13	0.34	0.10
В3	-2.78	0.21	1.35	0.23	В3	-2.78	0.21	0.81	0.22
B4	0.81	0.22	-2.10	0.26	B4	1.35	0.23	-2.10	0.26
B5	-0.46	0.08	-0.02	0.18	B5	-0.08	0.46	0.58	0.33
B6	0.06	0.08	0.20	0.16	B6	-0.22	0.35	-0.31	0.26
B7	0.05	0.04	0.06	0.06	В7	0.27	0.05	0.09	0.05
B8	0.16	0.07	0.04	0.08	B8	0.53	0.12	0.27	0.11
B9	0.39	0.07	0.06	0.12	B9	0.85	0.15	0.00	0.15

Table 11 Idealized cross-price elasticities and their standard deviations of B3 and B4 for the competitive condition: "all brands are offered at regular prices"

Impact on	Clout of				Impact	Receptivity of			
	B3		B4		of	B3		B4	
	Mean	SD	Mean	SD		Mean	SD	Mean	SD
B1	-0.26	0.05	1.44	0.11	B1	0.14	0.07	0.96	0.10
B2	0.32	0.06	0.41	0.04	B2	1.00	0.15	0.77	0.09
B3	-5.03	0.38	0.76	0.07	В3	-5.03	0.38	0.50	0.06
B4	0.50	0.06	-4.14	0.39	B4	0.76	0.07	-4.14	0.39
B5	-0.66	0.07	0.02	0.07	B5	-0.67	0.11	0.51	0.14
B6	-0.02	0.06	0.12	0.07	B6	0.25	0.09	-0.07	0.08
B7	-0.05	0.04	0.05	0.04	В7	0.61	0.07	0.31	0.06
B8	0.45	0.07	0.33	0.10	B8	1.07	0.11	0.66	0.14
B9	0.73	0.07	0.28	0.06	B9	1.91	0.18	0.51	0.09

prices. Although the corporate market share increases by 6% if B4 instead of B3 is price-promoted, the corporate market share when B3 and B4 are simultaneously on sale is even higher. The expected cumulated market share would be 37.79%. However, B3 and B4 will not gain as much share than if they were separately promoted, a clear indicator for cannibalistic effects. Now B3 would realize 14.19% compared to 20.57% and B4 can expect 23.60% compared to 26.33%. All competitors lose market share compared to the regular price condition.

In order to validate our results, we have performed a bootstrap analysis with 1000 random repli-

cations of our three-way array of cross-price elasticities and of our design matrix. The parameter estimates of the 1000 constrained TUCKALS3 solutions served to estimate mean idealized cross-price elasticities for the five a priori defined competitive conditions. They also served to estimate mean values of the market share predictions for the competitive conditions. The mean values (Mean) and their standard deviations (SD) for the selected competitive conditions are reported in Tables 8–12. A comparison of the bootstrap estimates with the estimates in Tables 3–7 reveals the high similarity of the estimated coefficients. The matrix correlation between the ide-

Table 12
Bootstrap idealized market-share estimates and their standard deviations for the a priori defined competitive conditions

	Competitive conditions										
	B3 on sale, B4 offered at regular prices		B4 on sale, B3 offered at regular prices		B3 and B4 are simultaneously on sale		All brands are offered at regular prices				
	Mean	SD	Mean	SD	Mean	SD	Mean	SD			
B1	5.77	1.60	5.16	0.63	2.40	0.73	8.05	1.14			
B2	8.46	1.22	9.60	1.57	7.26	1.64	12.19	2.13			
B3	18.70	11.25	5.41	2.93	14.18	1.95	4.27	0.45			
B4	5.13	1.02	26.27	15.42	23.71	3.56	8.48	0.65			
B5	8.22	0.87	10.78	0.98	18.13	8.30	11.38	1.55			
B6	3.71	1.22	2.85	0.49	1.46	0.52	3.52	0.70			
B7	30.13	8.58	17.06	2.98	15.96	3.87	24.79	2.01			
B8	7.19	4.75	6.99	1.86	7.99	1.82	10.24	2.07			
B9	12.68	5.31	15.88	3.05	8.90	2.18	17.08	1.85			
B3 + B4	23.83		31.68		37.89		12.75				

alized cross-price elasticities of the bootstrap estimates and the estimates of our model for the competitive condition "B3 on sale while B4 is offered at regular prices" is to 0.983. The matrix correlation between the idealized cross-elasticities for the competitive condition B4 on sale while B3 is offered at regular prices" is 0.999. It is 0.979 for the competitive condition "B3 and B4 are simultaneously on sale" and it is 0.998 for the competitive condition where all brands are offered at regular prices. The matrix correlation between our idealized market share predictions and the idealized market share predictions of the bootstrap analysis 0.9995. On the whole, the bootstrap analysis confirms the stability of our results and confirms our belief that the reported results are not sensitive to the definition of our design matrix.

In sum, the analysis of the expected market shares reveals that the brand management's pricing strategies for B3 and B4 may be optimal with respect to the cumulative market share. If B3 and B4 are price-promoted simultaneously they can increase the aggregate market share above those cumulative market shares where either B3 or B4 are price-promoted. However, though the combined price promotions of B3 and B4 are superior with respect to market shares they may not be optimal with respect to the profits. In order to solve this problem, the brand management has to consider the profit contributions of each brand when it is price-promoted and compare the profit outcomes across the three competitive conditions.

4. Conclusions

The methodology developed in this paper provides a means to analyze brand competition for certain marketing instruments. Our focus has been on price competition between corporate brands. Corporate brands are considered to be produced and marketed by the same company. Typically, consumer goods markets show corporate brands for almost all product categories. With respect to the marketing-mix-strategies for these brands, the brand management must estimate and evaluate the competitive interaction effects between its corporate brands in

order to develop and formulate effective marketing strategies.

Price competition can be established from an array of cross-price elasticities. These cross-price elasticities provide the necessary information to uncover the competitive interaction effects between brands. We have used an established market share attraction model to estimate the cross-price elasticities. The price elasticities are estimated for each time period, e.g. week. As such, the direct and cross-price elasticities of one particular period represent the competitive interaction effects of the brands for a certain competitive situation that characterize the time period, e.g. the price promotion of the market leader.

The cross-price elasticities across time form a three-mode three-way array that can be analyzed by a three-mode component analysis in order to detect the competitive structure of the market. However, this exploratory approach does not allow one to test and to estimate the competitive interaction effects for certain a priori defined competitive conditions. Thus, the price competition between corporate brands cannot fully be uncovered by the purely exploratory three-mode component analysis. For that reason, we have introduced the Constrained TUCKALS3-approach that enables us to investigate price competition between brands.

The Constrained TUCKALS3-approach is based on a reformulation and modification of the exploratory TUCKALS3-approach. It allows the integration of linear equality constraints on a subset of parameters and thus enables researchers to test a priori defined competitive conditions. In this way, the Constrained TUCKALS3-solution reveals interesting and meaningful idealized elasticity estimates and it also provides the information to estimate idealized market shares for certain a priori defined competitive conditions.

The empirical study of the price competition between the two corporate brands B3 and B4 clearly revealed cannibalistic effects between the two brands. With respect to the idealized price elasticities, the competitive strengths and weaknesses of the corporate brands could be established. The analysis has been conducted for several important competitive conditions for the two corporate brands. In particular, they were "B3 is on sale while B4 is offered at

regular prices", "B4 is on sale while B3 is offered at regular prices" and "B3 and B4 are simultaneously on sale". The results revealed managerially useful information about the competitive interactions effects between B3 and B4. The results also provided information regarding the strength and weaknesses of the corporate brands towards the prices of the other competing brands. In addition, we were able to compare these results with the idealized elasticities for the competitive condition in which no brand was offered at reduced prices. With respect to the idealized market shares, the empirical results revealed cannibalistic effects between the corporate brands in almost all a priori defined competitive conditions. However, the results also showed that simultaneous price reductions of B3 and B4 are with respect to the cumulative market share superior to price promotions in which only B3 or B4 is offered at reduced prices. Future research may also investigate temporal effects such the competitive cross-effects between brands that show regular pattern of promotions across time for example B3 is on sale and B4 is offered at regular prices in the current period while B4 is price-promoted and B3 is offered at regular prices in the subsequent period.

As discussed in Section 1, the presented research is related to the NEIO approach. The stated different forms of market conduct (independent, leader-follower and collusion) are typically investigated using game-theoretical approaches. Under independent behavior, each player takes its rival's strategic actions as given and wants to maximize its own profits (see Putsis, 1998). Under leader-follower behavior, one firm acts as the leader (i.e. it does not react to its rivals' actions) while its rivals follow changes in the leader's strategic behavior. Under collusive behavior, firms act to maximize joint profit. Kadivali et al. (1999) present a comparable definition of market conduct by classifying the competitive actions into independent, competitive and cooperative behavior. According to Ramaswamy et al. (1994), non-cooperative behavior can be consistent with retaliatory as well as with cooperative behavior. These models are specified by a system of demand and supply functions. In difference to this, our reduced form approach is only based on an attraction type demand function but it allows for multiplicative competitive effects of the marketing instruments. The market share of a specific brand is a function of its own attraction relative to the total attraction of all brands. The attraction of each brand can be operationalized in several different ways so that we can obtain a very flexible demand function with different interaction effects. Although the NEIO approach has been used increasingly with non-linear forms (e.g. Cotterill et al., 2000; Besanko et al., 1998), it allows only to test a very limited number of different forms of competitive interaction effects (e.g. Nash-behavior or Stack-elberg leader—follower behavior).

Another important feature of our approach compared to the NEIO approach is the possibility to investigate competition within differentiated product markets. The implementation of a NEIO model within differentiated product markets can be very cumbersome, if not impossible. The simultaneous estimation of a demand specification and a supply specification can in general only be realized by assuming linear demands due to the lack of closed form solutions using non-linear functional forms (see Besanko et al., 1998 or Chintagunta et al., 1999 for some exceptions). In addition, a typical NEIO analysis must usually be restricted to markets with only a few (two or three) products/brands (e.g. Gasmi et al., 1992). For differentiated product markets, Cotterill et al. (2000) (as one example) summarize the products into strategic groups (national brands vs. private labels) and estimate the competitive interactions between these strategic groups using flexible non-linear LA/AIDS demand functions and competitive reaction functions in order to account for the simultaneity of demand and supply. This research has also shown that the "pure" demand effects will be different than the measured effects that do not separate demand and competitive response.⁴ Where there are many players (brands or products) in the market, a brand-level discrete choice model on the demand side may be employed. However, in that case, the type of competitive interaction has to be a priori assumed as it cannot be estimated from the data (see e.g. Berry, 1994; Kadiyali et al., 1999). Thus, our reduced form approach still provides a valuable means to test and to investigate different competitive

⁴ We thank an anonymous reviewer for drawing our attention to this point.

conditions within differentiated product markets,⁵ although we acknowledge the simultaneity of demand and supply. The selection of a reduced form model therefore enables the user to model the demand side with an attraction model that also accounts for asymmetric competition. The attraction model is superior to (for example) linear functions in that it fulfills the logical consistency requirement (see Cooper and Nakanishi, 1988, p. 28 for a detailed discussion).

A further important feature of our approach is the possibility to simultaneously allow for different forms of competitive conditions (e.g. simultaneous price promotions, leader-follower behavior). In our empirical study, we test the cannibalism between brands B3 and B4 when they are exclusively or jointly price-promoted. This test and the corresponding selection of competitive conditions is only one of many possible ways to investigate the competitive interaction effects between these two brands. Tests may also include the analysis of possible leader-follower-structures in the market, for example B3 (B4) is on sale in the current period while B4 (B3) is on sale in the subsequent period or for example B3 and B4 are on sale in period t and are offered at reduced prices in the next period. In this respect, our approach allows for different competitive conducts across time, a fact that is very difficult to address in the NEIO approach. To date, only a few attempts within he NEIO approach have been made to model dynamics in competition (e.g. Kadiyali et al., 1999; Roberts and Samuelson, 1988; Slade, 1995; Vilcassim et al., 1999). In Kadivali et al.—for example two different NEIO models are estimated in order to investigate the effects of a product line extension on price competition. However, in this model line length is exogeneously determined. Chintagunta et al.—as another example—investigate in their NEIO approach dynamics within two periods. In contrast to these models, our approach enables us to investigate several different competitive conditions across time. Therefore, the model provides a valuable means to test dynamics of competition, a fact that is still a challenging task in the NEIO approach.

On the whole, the methodology introduced in this paper offers a means to investigate the price competition between brands in one product category. It enables brand management to detect and to analyze the competitive interaction effects between certain brands, e.g. the corporate brands or between the main competitors, for particular marketing instruments. The results provide managerially useful information for the development and improvement of marketing mix strategies.

Appendix A. The representation of competitive interaction effects

For the representation of the competitive interaction effects in any given market, it is appropriate to express the Tucker3-model in matrix notation by

$$X_t = A \sum_{r=1}^{R} c_{tr} G_r B' + E_t. \tag{A1}$$

where X_1 is the $(I \times J)$ -dimensional matrix of cross-elasticities in the tth time period. The $(P \times Q)$ -dimensional matrix G_r is the rth layer of the $(P \times Q \times R)$ -dimensional core array that contains the information how to link the P receptivity components with the Q clout components for the rth competitive condition. E_t is the $(I \times J)$ -dimensional component error matrix of the tth time period.

The approach of representing the competitive interactions in the market is mainly based on the information in the core array. From Eq. (4) or Eq. (5), it follows that the core array consists of three dimensions. Each of the R core slices corresponds to one competitive condition and indicates how to link the clout and the receptivity components of the other two component matrices. The information of the rth core slice (G_r) is used to represent the clout and the receptivity of each brand in a joint plot. The construction of joint plots is based on a singular value decomposition of the core array (see e.g. Kroonenberg, 1984). If we assume in this paper that I = J = the number of brands we get:

$$G_r = U \cdot D^2 \cdot V',$$

$$A^* = A \cdot U \cdot D,$$

$$B^* = B \cdot V \cdot D,$$
(A2)

with U and V representing the left and right singular vectors of G_r and D^2 as the diagonal matrix contain-

⁵ A notable exception may be seen in the studies of Cotterill (1994) and Cotterill et al. (2000) who have estimated residual demand elasticities in the soft drink market by combining LA/AIDS demand and price reaction functions.

ing the singular values of matrix G_{n} . The matrices and B^* are the coordinate matrices of the receptivity and clout of the brands in the joint plot of the rth competitive condition. In this competitive map, each brand is characterized by two vectors. One indicates the clout of the brand the other vector indicates the receptivity of the brand. Then the scalar products of the clout and the receptivity vectors of different brands measure the competitive interactions of the brands for the rth competitive condition. The corresponding idealized elasticities for certain competitive situations / conditions t^* are estimated by the procedure outlined in Tucker and Messick (1963). For any interesting pattern or week, the idealized elasticities are computed by summing over the weighted inner products:

$$X_{t^*} = \sum_{r=1}^{R} c_{t^*r} P_r. \tag{A3}$$

with X_{t^*} as the matrix of the corresponding idealized elasticities and P_r as the matrix of the inner products for the *r*th competitive condition:

$$P_r = AG_rB'. (A4)$$

The coefficients of the idealized elasticities may be interpreted directly or visualized in a competitive map by a singular value decomposition of the matrix X_{t^*} .

Kroonenberg and de Leeuw (1980) have provided a general method for the least squares fitting of the Tucker3 model, which minimizes the so-called TUCKALS3-loss function

TUCKALS3(A,B,C,G₁,...,G_R)

$$= \sum_{t=1}^{T} \left\| \boldsymbol{X}_{t} - \boldsymbol{A} \left(\sum_{r=1}^{R} c_{tr} \boldsymbol{G}_{r} \right) \boldsymbol{B}' \right\|^{2}$$
 (A5)

over column-wise orthonormal matrices A, B, C of the dimensionality $(I \times P)$, $(J \times Q)$, $(T \times R)$ and arbitrary matrices G_1, \ldots, G_R of order $(P \times Q)$. $\|\cdot\|$ denotes the Euclidean norm and X_t is the slice of the tth time period of the three-way elasticity array. The orthonormality constraint is only imposed on the estimation procedure (Kroonenberg, 1983). After convergence of the loss function, the component matrices can be exposed to any non-singular transformation matrix with the right rank.

Appendix B. The constrained parameter estimation of component matrix C

For the case of a constrained parameter estimation of matrix C, we get

$$f(C) = \|X - CG(A' \otimes B')\|^2 = \|X' - F'C'\|^2$$
$$= \sum_{t=1}^{T} \|x'_t - F'c_t\|^2$$
(B1)

with X as a $(T \times IJ)$ -dimensional matrix, x_t' as the $(IJ \times 1)$ -dimensional vector of the tth row of matrix X, G as the $(R \times PQ)$ -dimensional core array, F as the $(R \times IJ)$ -dimensional matrix $F = G(A' \otimes B')$ and c_t as the $(R \times 1)$ -dimensional vector of the tth row of matrix C.

Next the linear equality constraints are implemented into the estimation of certain parameters of component matrix C. Let d_t represent the $(R \times 1)$ -dimensional vector of the tth row of the design matrix D. Then we can get an estimate of the tth row of component matrix C by

$$c_t = ((F' * d_t)'(F' * d_t))^{-1}(F' * d_t)'X_t'$$
 (B2) with * as element-wise multiplication. In general $F' * d_t$ has no full column rank as these columns are completely filled with zeros when the corresponding design matrix element is zero. For this reason, the inverse matrix of $(F' * d_t)'(F' * d_t)$ cannot be calculated. In practical applications, the zero columns must be eliminated in a first step. After the estimation of the reduced c_t , the truncated elements of c_t

must be filled up with zeros again.

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