PSYCHOMETRIKA—VOL. 47, NO. 4.
DECEMBER, 1982

STATISTICAL EVALUATION OF MEASURES OF FIT IN THE LINGOES-BORG PROCRUSTEAN INDIVIDUAL DIFFERENCES SCALING

ROLF LANGEHEINE

INSTITUTE FOR SCIENCE EDUCATION AT THE UNIVERSITY OF KIEL

PINDIS, as recently presented by Lingoes and Borg [1978] not only marks the latest development within the scope of individual differences scaling, but, may be of benefit in some closely related topics, such as target analysis. Decisions on whether the various models available from PINDIS fit fallible data are relatively arbitrary, however, since a statistical theory of the fit measures is lacking. Using Monte Carlo simulation, expected fit measures as well as some related statistics were therefore obtained by scaling sets of 4(4)24 random configurations of 5(5)30 objects in 2, 3, and 4 dimensions (individual differences case) and by fitting one random configuration to a fixed random target for 5(5)30 objects in 2, 3, and 4 dimensions (target analysis case). Applications are presented.

Key words: individual differences scaling, procrustean transformations, matrix fitting, target analysis, configuration similarity, Monte Carlo evaluation.

Introduction

Investigators often wish to assess the similarity of two spatial representations of a set of objects. To cope with this problem, several essentially identical procedures for fitting one matrix to a given target have been developed [e.g., Cliff, 1966; Fischer & Roppert, 1965; Green, 1952; Kristof, 1964; Schönemann, 1966; Schönemann & Carroll, 1970]. During the sixties and seventies, considerable efforts were undertaken to extend the comparison of two spatial representations to the assessment of similarity among a set of N structures. In contrast to models incorporated into procedures such as INDSCAL [Carroll & Chang, 1970], ALSCAL [Takane et al., 1977] or COSPA [Schönemann et al., 1978, 1979], PINDIS (Procrustean INdividual DIfferences Scaling), as recently developed by Lingoes and Borg [1978], enables the researcher to give answers to questions such as

i) What is the structure common to N individuals, and, what degree of commonality may be realized among N individuals?
ii) To what degree can the common structure explain each individual structure?

It also offers a number of advantages over previous models:

i) PINDIS allows the researcher to choose among a maximum of five transformations or models, one admissible similarity transformation and four so-called inadmissible transformations (see next section): a dimensional salience model, a dimensional salience model with idiosyncratic orientation, a perspective model, and a perspective model with idiosyncratic origin.
ii) Since PINDIS proceeds in a stepwise manner, the hierarchies of transformations not only provide more insight into differences among individual structures, but allow one to evaluate the improvement in fit due to inadmissible transformations against the simpler transformations.

The author wishes to thank Ingwer Borg and Peter Schönemann, along with several anonymous reviewers, for a number of suggestions concerning this paper.

Requests for reprints and/or the volume of the simulation norms should be sent to: Rolf Langeheine, IPN at the University of Kiel, Olshausenstr. 40-60, D-2300 Kiel 1, Federal Republic of Germany.

0033-3123/82/1200-3040$00.75/0

iii) Inadmissible transformations may also be useful when fitting one matrix to a fixed target, which is a special case offered by the PINDIS program.

Though PINDIS has been applied in a variety of contexts such as social indicators research [Andrews & Inglehart, 1979; Borg & Bergermaier, 1979, 1981], business studies and the development of managerial skills [Maimon et al., 1980], perception [Borg, 1977; Borg & Lingoes, 1978; Coxon & Jones, 1980; Lingoes & Borg, 1978], attitude research [Borg & Lingoes, 1977; Feger, 1979; Lingoes & Borg, 1978], sociometry [Langeheine, 1978a, 1978b, 1979], test theory [Langeheine & Andresen, 1980], and educational evaluation [Langeheine, 1980], some researchers [e.g., Andrews & Inglehart, 1979; Coxon & Jones, 1980; Lingoes & Borg, 1978] have more or less explicitly stressed the lack of statistics for testing the significance of similarities between configurations. This problem has long been noticed with respect to fitting one matrix to a target [e.g., Korth & Tucker, 1975; Levine, 1977; Mulaik, 1972; Nesselroade & Baltes, 1970; Poor & Wherry, 1976]. In addition, the adequacy of the one available test by Poor & Wherry has recently been questioned by Borg [1981] and Borg and Bergermaier [1981] since the Poor and Wherry invariance index is not correct for distances.

The purpose of the present paper, therefore, is to report results from Monte Carlo studies covering both the PINDIS individual differences case and the case of fitting one matrix to a fixed target. Before describing the data-generating procedure, a brief description of what PINDIS does will be provided.

The Lingoes-Borg PINDIS Model

Method of Fitting

In contrast to other approaches, PINDIS starts from a set of configurations X_i $(i = 1, \ldots, N)$ of order $n \times m$ (objects $\times$ dimensions) pertaining to N individuals or other data sources. After norming all X_i's (centered at the origin) to unit length, an $n \times m$ centroid configuration Z is determined which is the average of all X_i's which have been optimally fitted to each other by an iterative procedure employing only admissible transformations (i.e., rotations/reflections, translations, and central dilations). The term "admissible" refers to those transformations which leave the observed comparative distances among the n objects invariant. Z thus has maximal average commonality with all X_i's. Each X_i is then optimally fitted to Z (target). The commonality is given by $r^2(\tilde{X}_i, Z)$, where $\tilde{X}_i$ refers to the fitted X_i (again by strictly admissible transformations only), and r^2 is the squared product moment correlation of the $n \times m$ coordinates of $\tilde{X}_i$ and Z. The average of the r^2's provides a measure for the degree of commonality Z shares with all $\tilde{X}_i$'s. This first commonality concept thus relates to the optimal similarity transformations of the X_i.

All further transformations use some distortion of Z (i.e., inadmissible transformations) in order to achieve a better approximation to the $\tilde{X}_i$. The first one of these transformations is the dimensional weighting of Z (dimensional salience model). Here, the dimensions of Z are stretched, shrunk, or reversed by negative weighting, to achieve a better fit to $\tilde{X}_i : r^2(\tilde{X}_i, Z^r W_i)$, where W_i is an $m \times m$ diagonal matrix of dimension weights and Z^r is the optimally rotated Z for *all* X_i's. A similar transformation defines the dimensional salience model with idiosyncratic orientation. It allows for an idiosyncratic rotation of Z to better correspond to each $\tilde{X}_i$. The efficacy is assessed by $r^2(\tilde{X}_i, Z_i^r W_i^r)$, where Z_i^r is a *subject-specific*, optimally rotated Z and W_i^r is the respective weight matrix. Both of these models have been used elsewhere, whereas the following two are specific to PINDIS. Let $r^2(\tilde{X}_i, V_i Z^t)$ indicate the success of fitting a row-weighted Z *to* $\tilde{X}_i$, where Z^t is an optimally translated Z for *all* X_i's and V_i is an $n \times n$ diagonal matrix of "vector" weights. This weighting conceives each object in Z as a terminus of a vector—emanating from the origin—that may be stretched, shrunk or even reversed in its direction. Finally, as Lingoes and Borg [1978] have shown, the predictability of $\tilde{X}_i$ may be further improved by an idiosyncratic translation of Z relative to each X_i: $r^2(\tilde{X}_i, V_i^t Z_i^t)$, where Z_i^t is a *subject-specific*, idiosyncratically translated Z and V_i^t is the associated matrix of vector weights. These last two models are called the perspective model and the perspective model with idiosyncratic origin.

In general, the problem reduces to minimizing the following loss function:

$$\operatorname{tr}(EE') = \min_{[R_i, S_i, W_i, V_i, k_i, t_i, u_i]}, \tag{1}$$

where

$$E = V_i(Z - jt_i')S_i W_i - k_i(X_i - ju_i')R_i. \tag{2}$$

X_i and Z in (2) refer to the individual and centroid configuration. W_i and V_i are diagonal matrices of dimension and vector weights, respectively. R_i and S_i denote orthonormal matrices for rotating X_i and Z. u_i and t_i are translation vectors, and j is the unit vector. Finally, k_i is a central dilation scalar. The different transformations outlined above therefore make use of different optimizations in (2). In the dimensional salience model, e.g., $V_i = I$, $S_i = S$ (an orthonormal rotation matrix used for all i) and t_i is a vector of zeros. It should be noted, however, that R_i, u_i, and k_i will generally be different under the various transformtions, resulting in different $\tilde{X}_i$'s which correspond to the outermost right-hand expression in (2).

In order to enable target fitting, the PINDIS program also provides an option for input of a fixed Z. In this case the process of generating Z will be bypassed, but the remaining procedure is the same as described above except that the idiosyncratic transformations are also omitted.

Relevance of Perspective Models

Whereas there seems to be no disagreement as to the psychological relevance of the dimensional salience models, this has been questioned for the perspective models—at least by one of our reviewers. We shall therefore briefly describe two cases where the perspective model may be useful in adequately mapping psychological phenomena. Feger [1979; cf. also Roskam, 1981] had subjects rate both the similarity (closeness) between a set of 9 objects (6 political parties and 3 institutions—trade unions, employer's association, church) and the closeness of the objects to the subject himself. Data were obtained from 9 subjects, with 12 replications each, within a period of 36 weeks. The 108 similarity matrices were first scaled nonmetrically in two dimensions and the resulting configurations were then submitted to PINDIS. In the resulting Z configuration, the origin was fixed at the point representing the subject. Intra- and interindividual differences could then be explained by shifting the object points radially with respect to this given point-of-view or perspective. Psychologically, the distance between the origin and each object point was interpreted as strength of preference, therefore establishing a simple relationship between the perceived similarity of the objects (represented by inter-object distance) and the subjects' preference for these objects. For an adequate interpretation the model is restricted to non-negative vector weights, however.

As a second example, consider the following situation from perceptual sociometry [Tagiuri, 1960]. In order to understand interpersonal relations, Tagiuri [1952] assesses two kinds of sociometric data: (a) person i's preferences for all of the other group members, and (b) i's perception of the preferences of the others towards himself. Later, Tagiuri

[1958] extended this approach to asking for all pairwise preferences of the group as perceived by each subject. Whereas Tagiuri used indices derived from these data in testing a variety of hypotheses, Feger [1977] translated Tagiuri's hypotheses and findings into statements about sets of distances. Feger thus focusses more clearly on structural aspects of the (perceived) sympathy structure(s) by contrasting the individual spaces with a (latent) common group space, e.g., the PINDIS Z derived from the individual configurations. Some of these statements may be combined into the following hypothesis: though we would assume that all individuals have basically the same perception of the group structure, individuals will perceive themselves more closely related to the central group members as compared with the group structure. This should especially hold for persons of low popularity (i.e., low sociometric status). In terms of PINDIS we would therefore expect a vectorially weighted Z to be in closer correspondence with the individual X_i's. So far, we know of no tests of these propositions.

We might further find that some other points in Z require substantial displacements, thus telling us that there is disagreement about who likes whom, or, more generally, about what goes with what. In this case, the vector weighting may be used as a procedure which provides indices about discontinuous relationships between Z and each X_i, respectively [Borg & Lingoes, 1977, 1978]. Subjects may merely perceive some objects differently relative to a reference system (Z) essentially shared by all of them [cf. also Coxon & Jones, 1980].

Data Generating Procedure

Despite some controversy on 2-way MDS Monte Carlo studies, different authors agree that results from this work can be useful for practical purposes since baseline fit measures obtained from scaling random data may be taken as a standard against which results obtained from actual data can be compared. The present study follows this rationale. It should be noted, however, that statistical testing is reduced to the rather weak hypothesis whether an actual r^2 exceeds one expected from scaling random configurations.

In order to cover ranges typically used in empirical studies it was decided to vary parameters relevant in PINDIS as follows: number of objects (O): 5(5)30; number of dimensions (D): 2, 3, and 4; number of configurations (N): 4(4)24. Starting with as few as 4 configurations is not only realistic but was mainly motivated by results of McCallum and Cornelius [1977] who were surprised about the negligible effect of number of individuals in their study on recovery of structure by ALSCAL. Nevertheless, they suspect that fewer than 15 individuals (their minimum) might have an effect. Though inclusion of as few as 5 objects is clearly unrealistic (at least in 3 and 4 dimensions) we decided to start with $O = 5$ to get some feeling of the PINDIS behaviour with a very small number of objects.

Though there seem to be some preferences as to the error model(s) for generating data in MDS simulations it is difficult to defend any particular method as the most plausible one for many psychological situations. In addition, Schönemann et al. [1979] noticed that the definition of randomness hardly did affect results. The distribution of their indices turned out to be nearly identical for data generated from three radically different parent distributions, i.e., uniform, standard normal and exponential. Indeed, this should not be surprising since random data are just random in structure irrespective of the parent distribution chosen if we consider the situation of "pure noise"—see Graef and Spence [1979] for the same conclusion when imposing varying degrees of error on the coordinates.

Given each combination of O, D, and N we therefore decided to sample coordinates of the X_i's from the computationally most simple distribution, i.e., the pseudo-random uniform on the interval 0 to 1, imposing the additional constraint that only those objects were retained that lie inside the unit hypersphere [Spence, 1972]. Each X_i was then orthogonalized according to the approach reported by Cohen and Jones [1974]. In order to remove the rather unrealistic situation of equal variance, dimensions were finally stretched or shrunk in a manner similar to that used by Weeks and Bentler [1979]: dimensions one through four were multiplied by 1.4, 1.2, 1.0 and 0.8, thus decreasing the variance as the dimensionality is increased.

For each combination of O, D, and N, 20 replications were run, thus resulting in a total of $(6)(3)(6)(20) = 2160$ analyses for the individual differences case. That is, expected fit measures reported herein pertain to (20) (N) X_i's and their centroid configurations, i.e., to at least 80 and at most 480 matches of the respective X_i's with their Z's. In each case, all 5 transformations were included. For the target fitting case, exactly the same X_i's were used in a second loop to fit $N - 1$ X_i's to a fixed target, X_1. That is, for *each* choice of O and D, a total of $(3 + 7 + 11 + 15 + 19 + 23)(20) = 1560$ fittings were done.

Results and Discussion

Detailed results (minimum, maximum, mean, standard deviation and 95% cut-off value of the distributions as well as standard deviation and 95% cut-off value of the difference distributions—both for individual fit measures and overall means and both in terms of r^2 and the respective Fisher-z's) are tabled in Langeheine [Note 1]. It is recommended to use these tables in evaluating actual results. The following presentation has to be limited to some general trends, all in terms of r^2.

Individual Differences Case

Figure 1 gives a compact picture for the similarity transformation. Without exception the fit decreases as N and O are increased and increases as D is increased. Similar patterns hold for both of the dimensional salience models, which, in general, do only slightly better than the similarity transform (cf. Figures 3 to 5).

With both of the perspective models, trends are the same for N and O as before, but, apart from $N = 4$ or $O = 5$, the fit decreases as D increases (cf. Figure 2 for the perspective model).

Finally, the grand means for O, N, and D (cf. Figures 3–5) clearly show that there is a considerable improvement in fit under both perspective models.

Some general trends from these analyses are evident:

i) r^2 decreases as the number of objects increases. This is in accord with results from previous 2-way MDS studies.

ii) r^2 decreases as the number of individuals increases. This decrement is much more pronounced for small N. Thus MacCallum and Cornelius' conjecture is confirmed that the number of individuals may have a considerable influence on the results. Due to the fact that coefficients are relatively stable for $N > 15$ their recommendation can be agreed upon: "Thus, it appears that empirical investigators can rest easily when they are unable to obtain large sample sizes ..." [MacCallum & Cornelius, 1977, p. 421], provided the risk of failing to observe individuals with possibly different structures seems acceptable.

iii) As to the effect of dimensionality there are two trends:

a) Given O and N, the following relationship holds without any exception for the similarity transformation and both of the dimensional salience models: r^2 in 2 dimensions $< r^2$ in 3 dimensions $< r^2$ in 4 dimensions. This, as well, corre-

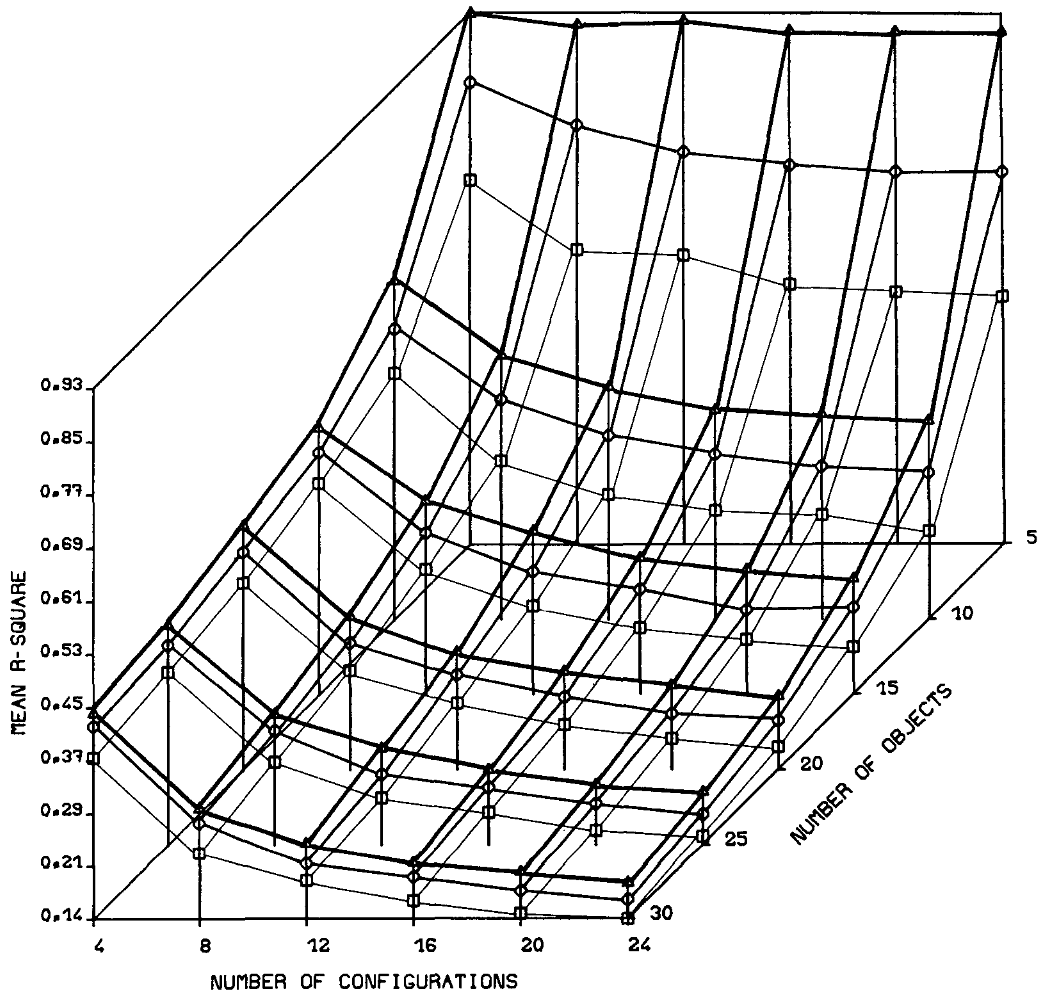

FIGURE 1
Similarity transformation. Mean r-square for number of configurations and number of objects in 2, 3, and 4 dimensions (□ :2-D, ○ :3-D, △ :4-D).

sponds with MacCallum and Cornelius and should not be surprising due to the greater freedom for motions in a higher dimensioned space.

b) Just the opposite effect was noticed for both of the perspective models, i.e.: r^2 in 2 dimensions $> r^2$ in 3 dimensions $> r^2$ in 4 dimensions, with the exception of some cases where $O = 5$ or $N = 4$. This result, too, would be expected since shifting single objects in the space to correspond better to a target by applying the same *one* vector weight to D dimensions should be easier if there are only two as compared to three or even four dimensions.

It should be kept in mind, however, that the analyses performed here used independent data for each choice of D. The effects noticed must, therefore, not necessarily hold for actual data which may have been scaled, say, in 2, 3, and 4 dimensions prior to processing them to PINDIS in common dimensionality.

iv) With respect to the dimensional salience models, it is interesting to note that, on the average, both of them result in only slightly better fit as compared to the

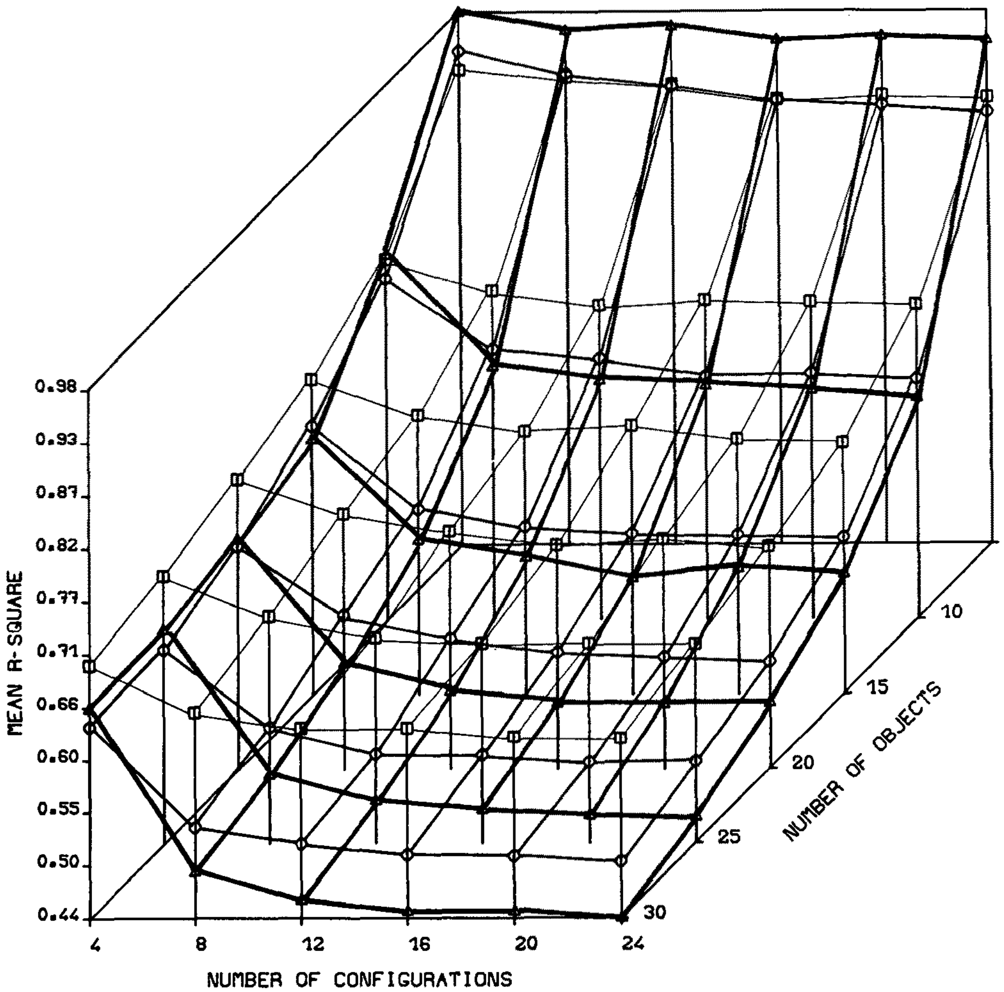

FIGURE 2
Perspective model. Mean r-square for number of configurations and number of objects in 2, 3, and 4 dimensions (□ :2-D, ○ :3-D, △ :4-D).

similarity transformation. This might be suspected for most sets of empirical data as well, which would merely indicate that these models are not adequate—though they are frequent in the literature. Results of this study therefore enable users to test whether improvement in fit is significant to claim individual differences due to dimensional weighting.

v) Under the perspective models, on the other hand, we can generally notice an enormous improvement in fit. Users of PINDIS should therefore carefully consider whether the gain in fit is worth the cost of the additional parameters instead of using arbitrary rules of thumb. For a reevaluation of some previously published results, see Langeheine [Note 1] and the final section.

vi) With respect to as few as 5 objects it is evident that the only realistic case to be considered would be that of 2 dimensions, restricted to the similarity and dimensional salience transformations.

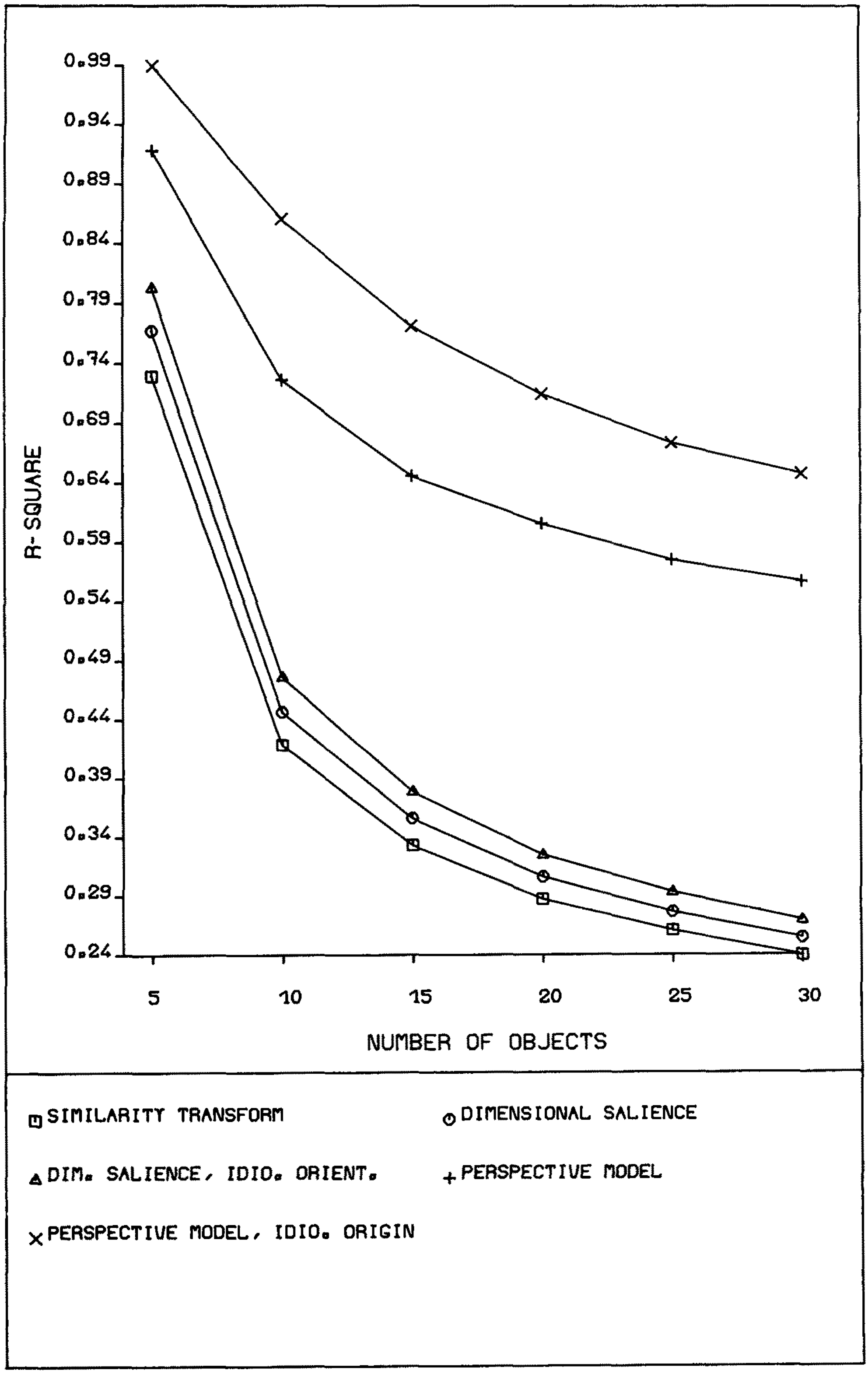

FIGURE 3
Grand mean r-square for number of objects.

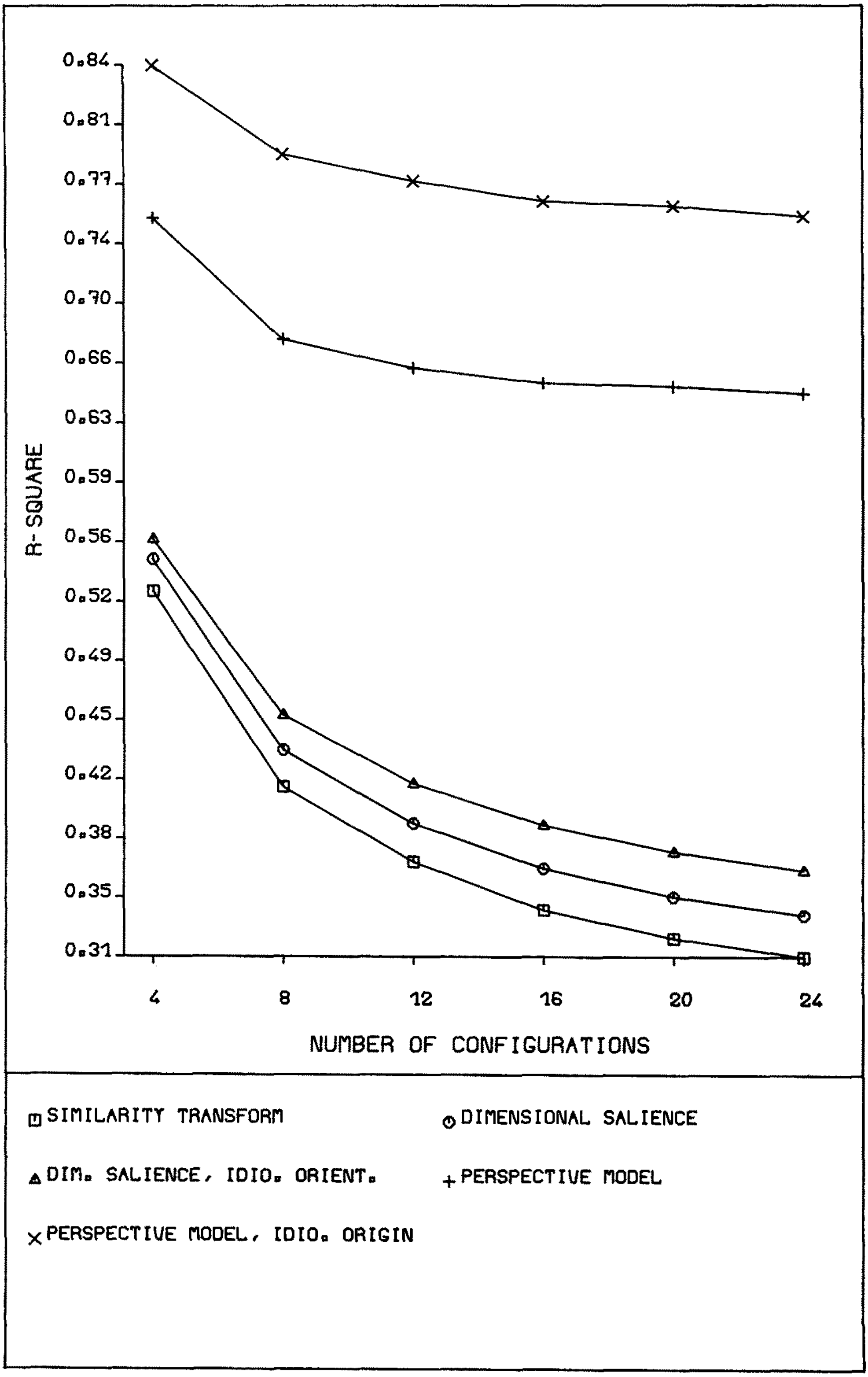

FIGURE 4
Grand mean r-square for number of configurations.

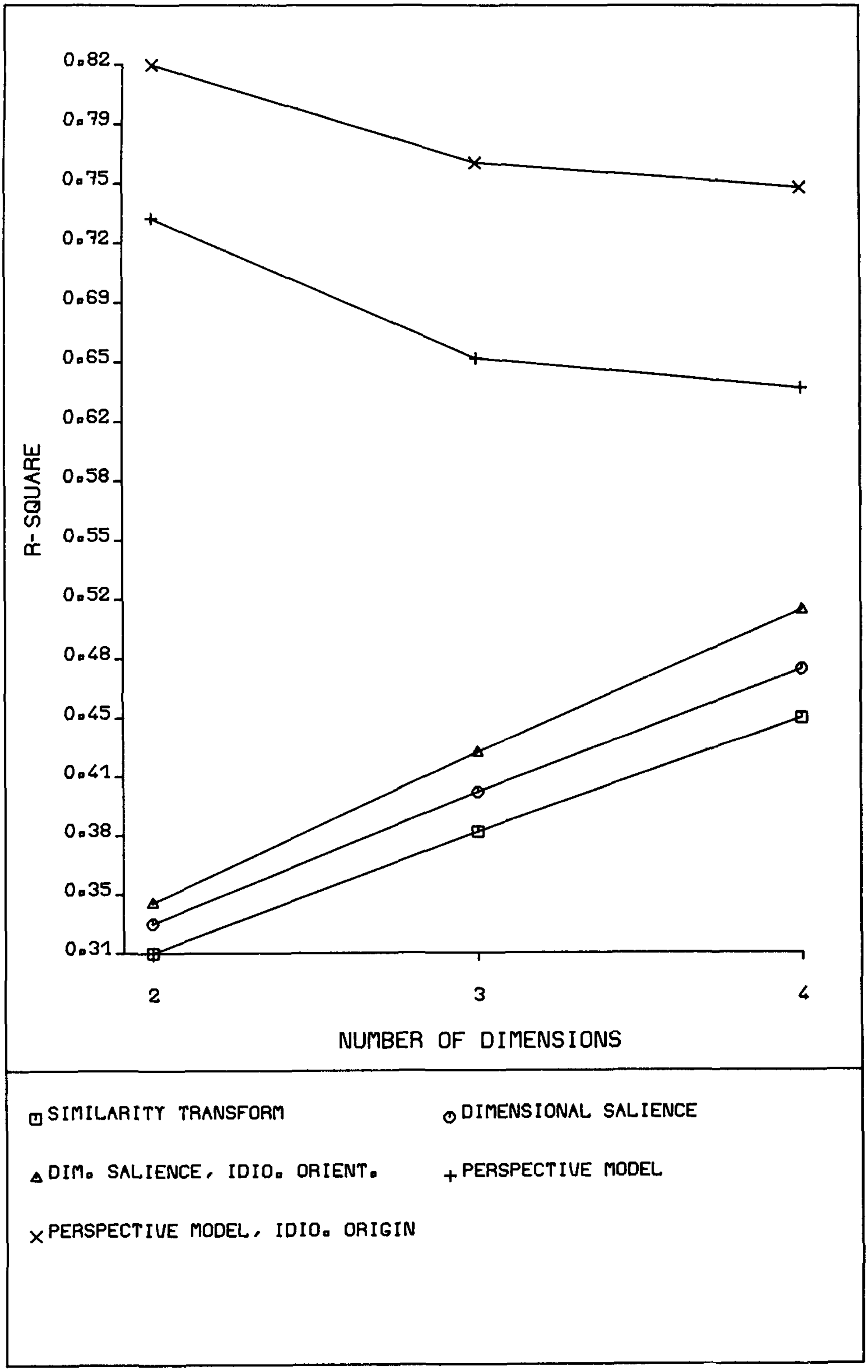

FIGURE 5
Grand mean r-square for number of dimensions.

Fitting One Configuration to a Fixed Target

Results, all in terms of grand mean r^2's from 1560 coefficients, are presented in Figure 6. As to the effects of number of objects, number of dimensions and transformations, these correspond exactly to those noticed in the individual differences case. The only irregularity noticeable is, that, in case of 5 objects, the perspective model does better in 4 as compared to 3 and 2 dimensions.

Though PINDIS has been mainly developed to handle the individual differences case, the option of fitting one configuration to a fixed target may be useful in a variety of cases. Some of these are: comparison of results obtained from metric and monotone distance analysis [2-way MDS, cf. Weeks & Bentler, 1979]; comparison of results obtained from different 3-way MDS procedures, say, INDSCAL and PINDIS; comparison of some structure matrix set up according to theoretical considerations with some obtained matrix. Be it in the area of MDS or factor analysis, the latter case offers not only one sort of confirmatory analysis but the possibility to fit the obtained matrix to an optimal target. In view of the criticism on the Poor and Wherry [1976] invariance index [Borg, 1981; Borg & Bergermaier, 1981] we rather recommend to use the current norms instead of the Poor and Wherry test in evaluating configurational similarity of two matrices.

Applications

For actual data, there are two questions to answer:

i) When should fit measures obtained for any one single transformation be judged to exceed those expected by chance?
ii) When should improvement in fit for a more complex transformation be considered significant as compared to a hierarchically less complex one? In general, we are interested in the following comparisons: 2 vs. 1, 3 vs. 1, 4 vs. 1, 5 vs. 1, 3 vs. 2 and 5 vs. 4 (where 1 to 5 refer to the transformations according to their complexity, i.e., similarity transformation, dimensional weighting, dimensional weighting with idiosyncratic rotation, perspective model, perspective model with idiosyncratic origin, respectively).

In order to answer these questions, different strategies have been examined in Langeheine [Note 1] with the conclusion that results were nearly identical when the 95% cut-off values (percentiles) of the random coefficient distributions (as well as those of the difference distributions) or the respective mean and standard deviation were used for Fisher-z's on r of the PINDIS r^2 (which is the squared produce moment correlation of the $n \times m$ coordinates of $\tilde{X}_i$ and Z). We shall therefore use the 95% cut-off values in terms of Fisher-z's in evaluating the fit of single transformations as well as improvement in fit for a more complex transformation.

Example 1

Among other things, Maimon et al. [1980] used PINDIS to evaluate the hypothesis that structures of the interrelationships among 9 skills required for the performance of managerial skills are similar for 4 groups. Table 1 contains observed Fisher-z's corresponding to their results as well as 95% cut-off values obtained by using the PINDIS simulation program with the exact parameters ($O = 9$, $D = 2$, $N = 4$). Considering the first 3 transformations, Maimon et al. conclude that the 4 groups differ in degree of similarity to the centroid. On the other hand, they claim support for their hypothesis since the "... vector weighting ... results in a very strong pattern of similarity" [Maimon et al., 1980, p. 739].

Results from Table 1 make evident that we disagree with these conclusions, at least in part. Under the similarity transformation (column 1), only group 1 exceeds the critical

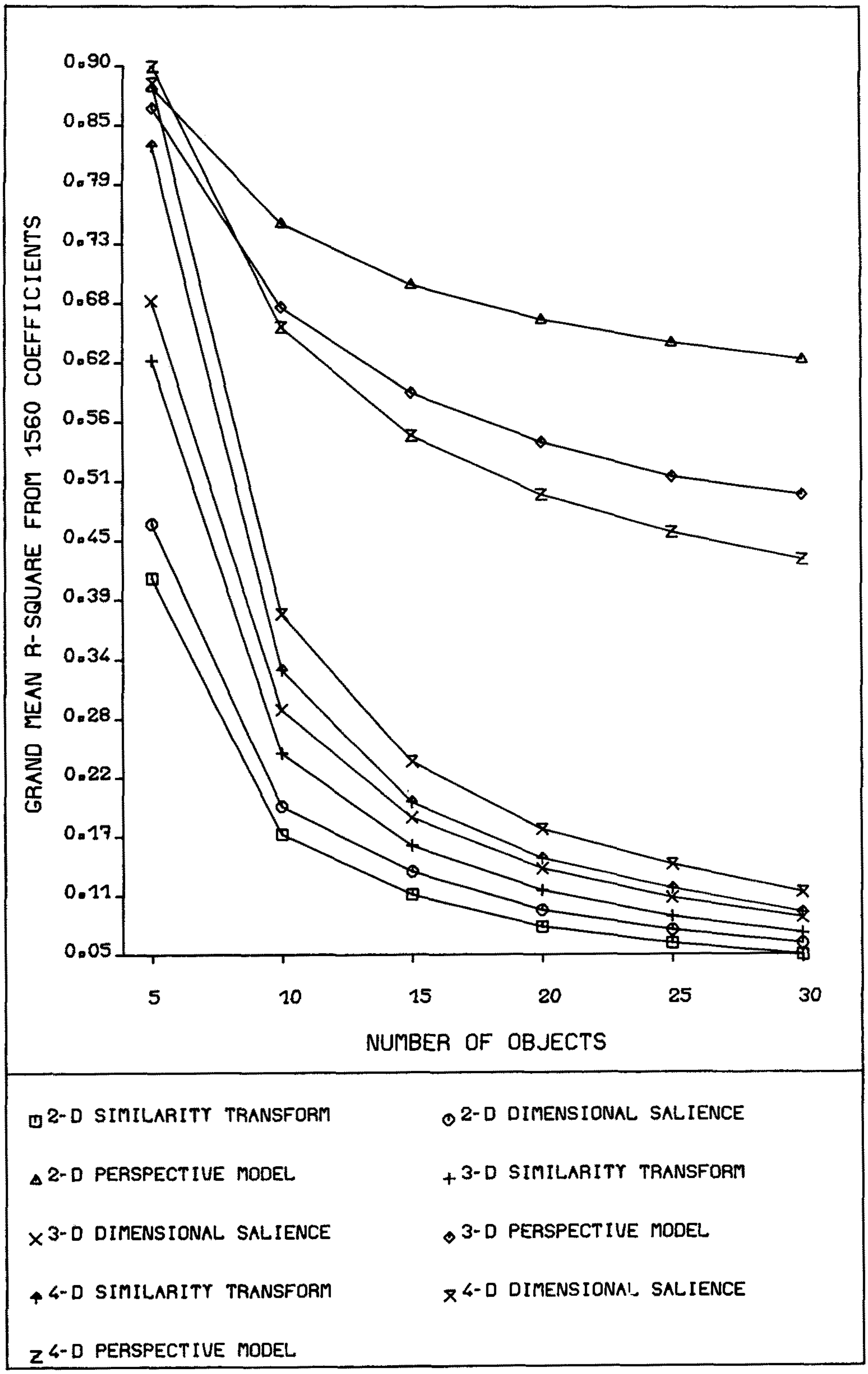

FIGURE 6
Empirical values for fitting one matrix to a fixed target.

Table 1

Maimon et al. Data

Group	Transformation 1	2	3	4	5
	observed fit in terms of Fisher-z's				
1	1.502*	1.567	1.567	2.005	2.439
2	.799	.881	.969	1.818	2.005
3	.939	.984	1.000	1.818	2.644
4	1.016	1.032	1.099	1.602	2.993*
mean	1.064	1.116	1.159*	1.811	2.520
	95% cut-off Fisher-z's from simulation				
indiv.	1.228	1.255	1.258	2.175	2.923
mean	1.028	1.098	1.102	1.961	2.536

	95% cut-off Fisher-z differences from simulation comparison of transformation					
	2 vs. 1	3 vs. 1	3 vs. 2	4 vs. 1	5 vs. 1	5 vs. 4
indiv.	.067	.080	.038	1.208	1.867	1.210
mean	.041	.057	.026	.967	1.538	.834

Note: Transformation: 1= similarity transformation, 2= dimensional weighting, 3= dimensional weighting with idiosyncratic rotation, 4= perspective model, 5= perspective model with idiosyncratic origin

indiv.= individual fit measures, mean= overall mean

*transformation accepted (due to significant single z and significant improvement in fit for inadmissible transformation)

value. With the exception of the mean, all of the dimensional weightings (columns 2 and 3) fall short of the respective critical values. This holds as well for the perspective model (column 4) though the corresponding r^2's as well as gain in fit over the similarity transformation might appear to be quite impressive in 3 of 4 cases as well as in overall terms (mean). Group 4 is the only one for which an idiosyncratically translated and vectorially weighted Z (column 5) fits the respective X_i better than would be expected by chance, both in terms of the single z and in improvement in fit. This would indicate, however, that certain points in Z have to be considerably relocated in order for Z to match X_4, thus indicating an individual *difference*. On the whole, we can thus only confirm that there are considerable differences among the 4 structures.

Example 2

As a second example, we shall take a closer look at results of fitting 6 15 × 3 configurations to a fixed target, as reported by Coxon and Jones [1980]. If the analysis would have been performed by any procedure restricted to the similarity transformation, our conclusion would have been that only subjects 1, 5, and 6 share some aspects with the target (cf. Table 2). The latter two cases illustrate as well that the vector weighting (transformation 4) by no means guarantees "to make silk purses from sow's ears" [Coxon & Jones, 1980, p. 65]. Since Coxon and Jones used rules of thumb they obviously overlooked that a dimensionally weighted target fits X_1 in excess of chance. As to subjects 3

Table 2

Coxon & Jones Data

Subject	Observed and 95% cut-off z's for transformation** 1	2	4
1	.667	.772*	1.317
2	.438	.509	1.134
3	.327	.378	1.390*
4	.495	.522	1.444
5	.881*	.939	1.081
6	.812*	.826	1.081
95% cut-off z***	.539	.601	1.247

95% cut-off z differences*** comparison of transformation

2 vs. 1	4 vs. 1
.103	.908

* Transformation accepted (cf. Table 1)

** cf. Table 1

*** cf. Langeheine [Note 1, Appendix A3-5]

Table 3

Lingoes & Borg Data

Subject	Observed and 95% cut-off z's for transformation** 1	2	4
1	1.773*	1.935	2.375
2	1.880	2.168*	2.120
3	.795	1.511*	1.370
4	2.111	2.406*	2.573
5	.876	1.326*	1.380
95% cut-off z***	.679	.741	1.812

95% cut-off z differences*** comparison of transformation

2 vs. 1	4 vs. 1
.172	1.445

* Transformation accepted (cf. Table 1)

** cf. Table 1

*** cf. Langeheine [Note 1, Appendix A3-1] and 4, finally, we agree with their conclusion that both have something in common with the target, i.e., the dimensions, though not the points which require considerable replacements [cf. Coxon & Jones for some details].

Example 3

Lingoes and Borg [1978] reanalyzed the Helm color data by deriving a Z from the 11 normal subjects and then relating the 5 color deficients' spaces to this fixed Z. Table 3 shows the respective observed z's and cut-off values. With the exception of subject 1, the theoretical expectation is confirmed that the dimension weighting (but not the vector weighting) improves the fit significantly. The gain in fit turns out to be most impressive for subjects 3 and 4 as is already obvious from the respective r^2's. Note that we would have to reject the null hypothesis of no similarity in all 5 cases if the analysis had been restricted to the similarity transform only.

REFERENCE NOTE

1. Langeheine, R. Approximate norms and significance tests for the Lingoes-Borg procrustean individual differences scaling (PINDIS). Research Report No. 39. Kiel, Institute for Science Education, 1980.

REFERENCES

Andrews, F. M. & Inglehart, R. F. The structure of subjective well-being in nine western societies. *Social Indicators Research*, 1979, *6*, 75–90.

Borg, I. Geometric representations of individual differences. In J. C. Lingoes, E. E. Roskam, & I. Borg (Eds.), *Geometric representations of relational data. Readings in multidimensional scaling.* Ann Arbor: Mathesis Press, 1979.

Borg, I. *Anwendungsorientierte Multidimensionale Skalierung.* Berlin: Springer, 1981.

Borg, I. & Bergermaier, R. Die Ähnlichkeit von Einstellungsstrukturen zur Lebensqualität in elf westlichen Gesellschaften. *Zeitschrift für Sozialpsychologie*, 1979, *10*, 253–261.

Borg, I. & Bergermaier, R. Some comments on "The structure of subjective well-being in nine western societies" by Andrews & Inglehart. *Social Indicators Research*, 1981, *9*, 265–278.

Borg, I. & Lingoes, J. C. Ein direkter Transformationsansatz der multidimensionalen Analyse dreimodaler Datenmatrizen: Theorie und Anwendungen. *Zeitschrift für Sozialpsychologie*, 1977, *8*, 98–114.

Borg, I. & Lingoes, J. C. What weights should weights have in individual difference scaling? *Quality and Quantity*, 1978, *12*, 223–237.

Carroll, J. D. & Chang, J. J. Analysis of individual differences in multidimensional scaling via an N-way generalization of "Eckart-Young" decomposition. *Psychometrika*, 1970, *35*, 283–319.

Cliff, N. Orthogonal rotation to congruence. *Psychometrika*, 1966, *31*, 33–42.

Cohen, H. S. & Jones, L. E. The effects of random error and subsampling of dimensions on recovery of configurations by non-metric multidimensional scaling. *Psychometrika*, 1974, *39*, 69–90.

Coxon, A. P. M. & Jones, C. Multidimensional scaling: Exploration to confirmation. *Quality and Quantity*, 1980, *14*, 31–73.

Feger, H. Quantitative sociometry: Problems, methods and first results. In H. J. Hummell & R. Ziegler (Eds.), *Anwendung mathematischer Verfahren zur Analyse sozialer Netzwerke.* Duisburg: Verlag der Sozialwissenschaftlichen Kooperative, 1977.

Feger, H. Einstellungsstruktur und Einstellungsänderung: Ergebnisse, Probleme und ein Komponentenmodell der Einstellungsobjekte. *Zeitschrift für Sozialpsychologie*, 1979, *10*, 331–349.

Fischer, G. & Roppert, J. Ein Verfahren der Transformationsanalyse faktorenanlytischer Ergebnisse. In J. Roppert & G. Fischer (Eds.), *Lineare Strukturen in Mathematik und Statistik.* Wien: Physika-Verlag, 1965.

Graef, J. & Spence, I. Using distance information in the design of large multidimensional scaling experiments. *Psychological Bulletin*, 1979, *86*, 60–66.

Green, B. F. The orthogonal approximation of an oblique structure in factor analysis. *Psychometrika*, 1952, *17*, 429–440.

Korth, B. & Tucker, L. R. The distribution of chance congruence coefficients from simulated data. *Psychometrika*, 1975, *40*, 361–372.

Kristof, W. Die beste orthogonale Transformation zur gegenseitigen Überführung zweier Faktormatrizen. *Diagnostica*, 1964, *10*, 87–90.

Langeheine, R. The evaluation of classroom social structure by three-way multidimensional scaling of sociometric data. *Studies in Educational Evaluation*, 1978a, *4*, 185–208.

Langeheine, R. Strukturelle Stabilität—Ein neuer Ansatz zur Überprüfung der Reliabilität in der Soziometrie. *Psychologische Beiträge*, 1978b, *20*, 571–588.

Langeheine, R. *Strukturanalytische Untersuchungen der Schulklasse. Eine Analyse mit Hilfe multidimensionaler Skalierungsmodelle.* Frankfurt: Lang, 1979.

Langeheine, R. Structural aspects reconsidered: A reanalysis of two studies in educational evaluation. *Studies in Educational Evaluation*, 1980, *6*, 329–341.

Langeheine, R. & Andresen, N. Strukturelle Stabilität des AFS. *Zeitschrift für Empirische Pädagogik*, 1980, *4*, 203–212.

Levine, M. S. *Canonical analysis and factor comparison*. Beverly Hills: Sage, 1977.

Lingoes, J. C. & Borg, I. A direct approach to individual differences scaling using increasingly complex transformations. *Psychometrika*, 1978, *43*, 491–519.

MacCallum, R. C. & Cornelius III, E. T. A Monte Carlo investigation of recovery of structure by ALSCAL. *Psychometrika*, 1977, *42*, 401–428.

Maimon, Z., Venezia, I. & Lingoes, J. C. How similar are different results? *Quality and Quantity*, 1980, *14*, 727–742.

Mulaik, S. A. *The foundations of factor analysis*. New York: McGraw-Hill, 1972.

Nesselroade, J. R. & Baltes, P. B. On a dilemma of comparative factor analysis: A study of factor matching based on random data. *Educational and Psychological Measurement*, 1970, *30*, 935–948.

Poor, D. D. S. & Wherry, R. J. Invariance of multidimensional configurations. *British Journal of Mathematical and Statistical Psychology*, 1976, *29*, 114–125.

Roskam, E. E. Contributions of multidimensional scaling to social science research. In I. Borg (Ed.), *Multidimensional data representations: When and why*. Ann Arbor: Mathesis Press, 1981.

Schönemann, P. H. A generalized solution to the orthogonal procrustes problem. *Psychometrika*, 1966, *31*, 1–10.

Schönemann, P. H. & Carroll, R. M. Fitting one matrix to another under choice of a central dilation and a rigid motion. *Psychometrika*, 1970, *35*, 245–255.

Schönemann, P. H., James, W. L. & Carter, F. S. COSPA: Common space analysis—A program for fitting and testing Horan's subjective metric model. *Journal of Marketing Research*, 1978, *15*, 268–272.

Schönemann, P. H., James, W. L. & Carter, F. S. Statistical inference in multidimensional scaling: A method for fitting and testing Horan's model. In J. C. Lingoes, E. E. Roskam & I. Borg (Eds.), *Geometric representations of relational data. Readings in multidimensional scaling.* Ann Arbor: Mathesis Press, 1979.

Spence, I. A Monte Carlo evaluation of three nonmetric multidimensional scaling algorithms. *Psychometrika*, 1972, *37*, 461–486.

Tagiuri, R. Relational analysis: An extension of sociometric method with emphasis upon social perception. *Sociometry*, 1952, *15*, 91–104.

Tagiuri, R. Social preference and its perception. In R. Tagiuri & L. Petrullo (Eds.), *Person perception and interpersonal behavior*. Stanford: Stanford University Press, 1958.

Tagiuri, R. Perceptual sociometry. In J. L. Moreno (Ed.), *The sociometry reader*. New York: The Free Press, 1960.

Takane, Y., Young, F. W. & De Leeuw, J. Nonmetric individual differences multidimensional scaling: An alternating least squares method with optimal scaling features. *Psychometrika*, 1977, *42*, 7–67.

Weeks, D. G. & Bentler, P. M. A comparison of linear and monotone multidimensional scaling models. *Psychological Bulletin*, 1979, *86*, 349–354.

Manuscript received 11/10/80
First revision received 7/6/81
Final version received 5/21/82