PSYCHOMETRIKA—VOL. 46, NO. 1.
MARCH, 1981

THE EXTENSION OF COMPONENT ANALYSIS TO FOUR-MODE MATRICES

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A model for four-mode component analysis is developed and presented. The developed model, which is an extension of Tucker's three-mode factor analytic model, allows for the simultaneous analysis of all modes of a four-mode data matrix and the consideration of relationships among the modes. An empirical example based upon viewer perceptions of repetitive advertising shows the four-mode model applicable to real data.

Key words: advertising, component analysis, factor analysis, four-mode models, three-mode models.

Tucker's [1963, 1964, 1966] three-mode factor analysis has given behavioral researchers a tool for analyzing metric data arranged or cross-classified in a three-mode matrix. A frequently mentioned example of data appropriate for three-mode analysis is that often obtained with Osgood, Suci and Tannenbaum's [1957] semantic differential. Indeed, ratings from individuals as to the meaning of several concepts in terms of semantic differential scales have frequently been analyzed with the three-mode model [e.g., Gritin, 1970; Hentschel & Klintman, 1974; Muthen et al., 1977; Snyder & Wiggins, 1970; Tzeng, 1976]. In such semantic differential studies, the individual mode × concept mode × scale mode data cube was decomposed into basic components corresponding to each of the three modes as well as an internal core matrix. The core matrix in such a three-mode study represents a data cube, like the original semantic differential data, except that its order corresponds to the ranks of the modes in the original cube of data.

Higher-order data matrices of four or more modes of classification, however, can also exist. For example, a consumer psychologist could have a sample of individuals use semantic-differential scales (good-bad, slow-fast, heavy-weak, etc.) to indicate their perceptions of different meat products (hot dogs, steak, bologna, etc.) within a set of diverse consumption situations (when hungry and watching TV alone late at night, when hosting a dinner party for a few good friends, when dining with your family at home on a weekday, etc.). In such a consumer behavior study, an individual × scale × meat product × situation four-mode data matrix would be collected. Although Tucker has suggested extending component analysis to four or more modes of data classification, the meaning or structure of a four-mode model has not been articulated. Through exposition of a mathematical extension of Tucker's model and an empirical example, this paper will make clear the structure of a four-mode component model.

This research was supported by the University of Kansas School of Business Research Fund provided by the Fourth National Bank & Trust Company, Wichita. The ideas and opinions expressed herein are solely those of the author.

The author acknowledges with thanks Deborah A. Aycox, for collecting the data analyzed in the empirical example, and John S. Brennan, for writing matrix algebra computer subroutines used to perform the data analysis.

Requests for reprints and copies of raw data upon which the reported analysis is based should be sent to John L. Lastovicka, School of Business, University of Kansas, Lawrence, Kansas 66045.

0033-3123/81/0300-2959$00.75/0

A Model for Four-Mode Component Analysis

The four-mode component model can be applied to a four-mode data matrix Y which has cell entries $y_{i'j'k'l'}$ such that $i' = 1, 2, 3, \ldots, i, j' = 1, 2, 3, \ldots, j, k' = 1, 2, 3, \ldots, k$, and $l' = 1, 2, 3, \ldots, l$. Such data would be collected in the consumer behavior meat product study described in the last paragraph. In short, the data appropriate for four-mode component analysis consists of a matrix of metric data whose elements are identified by four modes of classification.

The fundamental four-mode component analysis model can be written as:

$$y_{i'j'k'l'} \cong \hat{y}_{i'j'k'l'} = \sum_{m'=1}^{m} \sum_{p'=1}^{p} \sum_{q'=1}^{q} \sum_{r'=1}^{r} a_{i'm'} b_{j'p'} c_{k'q'} d_{l'r'} g_{m'p'q'r'} . \tag{1}$$

The data, the $y_{i'j'k'l'}$'s, are modeled by approximations, the $\hat{y}_{i'j'k'l'}$'s. The intent is that discrepancies between the data and the approximations are small. The approximations are based on four modes (m, p, q, and r) which are thought to be conceptually more basic than the modes employed in collecting the original data. Each of these more basic modes corresponds to one of the modes in the original data matrix Y: m corresponds to i, p to j, q to k and r to l. This four-mode component analysis model, like all component models, is essentially a data reduction technique. Therefore, it is hoped that $m \ll i$, $p \ll j$, $q \ll k$, and $r \ll l$. Each of the basic modes represents the number of components in the domain of the corresponding mode in the original data.

The coefficients $a_{i'm'}$, $b_{j'p'}$, $c_{k'q'}$ and $d_{l'r'}$ are elements in component loading matrices ${}_iA_m$, ${}_jB_p$, ${}_kC_q$ and ${}_lD_r$, respectively. For example, if the i mode in a four-mode data matrix represents responses to a set of i questionnaire items, the coefficient $a_{i'm'}$ would represent the loading of question i' on the latent component m'. Alternatively, it *may* be proper to interpret the basic modes as idealized entities [Helm & Tucker, 1962; Tucker & Messick, 1960].

The coefficients $g_{m'p'q'r'}$ are elements in the four-mode matrix G, which following Tucker, is termed the core matrix. The core matrix describes the interrelationships between the components found in each of the four modes. In a sense, the core matrix can be thought of as a set of component scores. In the original four-mode data matrix, Y, each element represents a particular cross-classification of some combination of each of the i, j, k and l modes. In the same way, every element in the core matrix, G, represents a particular cross-classification of each of the m, p, q and r basic modes.

The model specified in (1) is not a factor analytic model in the sense of the classical common factor analytic model which contains common and unique factors. The model in (1) does not provide for unique factors. It therefore is best labeled a four-mode component model even though the Tucker three-mode model, which is also a component model, is referred to as a factor analytic model.

Using Tucker's notion of a combinatorial mode or "the Cartesian product of two [or more] elementary modes . . . denoted by the letters of the two [or more] elementary modes . . ." [Tucker, 1966, p. 281], a four-mode matrix can be represented in two dimensions with a common two-mode matrix form such that one mode of the two-mode matrix form is a combinatorial mode. Thus, the four-mode matrix Y with entries $y_{i'j'k'l'}$ could be represented many ways in two dimensions including ${}_iY_{jkl}$, ${}_jY_{ikl}$, or ${}_lY_{ijk}$ where i, j, or l, respectively, are the row orders and jkl, ikl or ijk are the respective combinatorial mode column orders. The combinatorial mode jkl may be interpreted like variable subscripts initialized by do-loops in Fortran computer programs. That is, j is the outermost loop which changes the least quickly, k is the next inner loop which changes more quickly than i but not as fast as l, and l is the innermost loop changing more quickly in value than either j or k.

By relying upon combinatorial modes, the model in (1) can be written in matrix form as:

$$ {}_iY_{jkl} \cong {}_i\hat{Y}_{jkl} = {}_iA_m\, G_{pqr}[({}_pB_l \otimes {}_qC_k) \otimes {}_rD_l], \tag{2} $$

where $\otimes$ represents the Kronecker or direct product of two matrices. (The Kronecker product, ${}_pB_j \otimes {}_qC_k$, produces a supermatrix of order pq by jk containing submatrices ${}_qC_k$ scaled by the elements of matrix ${}_pB_j$.) Alternatively, by representing matrix Y in the ${}_jY_{ikl}$ form, (1) could be written in matrix form as:

$$ {}_jY_{ikl} \cong {}_j\hat{Y}_{ikl} = {}_jB_p\, G_{mqr}[({}_mA_i \otimes {}_qC_k) \otimes {}_rD_l]. \tag{3} $$

Due to the different ways of representing a four-mode data matrix with combinatorial modes, there are many matrix representations of the fundamental four-mode component model. In each matrix representation of the four-mode model the basic modes used for the rows and columns of matrix G are transformed by the appropriate loadings matrix to the corresponding modes in the original data.

The ranks of the four-mode data matrix, Y, when written in different two-dimensional representations are not necessarily the same. That is, the ranks of ${}_iY_{jkl}$, ${}_jY_{ikl}$, ${}_kY_{ijl}$, and ${}_lY_{ijk}$ are not necessarily equal. The ranks, m, p, q, and r, are restricted by a group of inequalities stating that no one of the four ranks can be greater than the product of the remaining three ranks. For justification, consider the core matrix and four possible ways of representing it in two dimensions with combinatorial modes: ${}_mG_{pqr}$, ${}_pG_{mqr}$, ${}_qG_{mpr}$, and ${}_rG_{mpq}$. In each case, the number of rows in each form of G equals the rank of Y written in the corresponding manner of combinatorial modes (e.g., rank $({}_iY_{jkl}) = m$). The number of columns of each form of G is the product of the ranks of Y written in remaining noncorresponding manners (e.g., rank $({}_jY_{ikl}) \times$ rank $({}_kY_{ijl}) \times$ rank $({}_lY_{ijk}) = p \times q \times r$). For a moment, consider a violation of the required rank relationship such that the number of rows of G (written in any of the four ways) were greater than the number of columns. In this situation, the rank of that way of representing G could not be the number of rows but would necessarily be a smaller value equal to or less than the number of columns in G (as the upper limit of the rank of a matrix is the smaller of the two values used to specify matrix order). Therefore, no one of the four values m, p, q, or r can be greater than the product of the remaining three.

By computing a set of cross-product matrices:

$$ {}_iY_{jkl}Y_i = {}_iM_i, \tag{4} $$

$$ {}_jY_{ikl}Y_j = {}_jP_j, \tag{5} $$

$$ {}_kY_{ijl}Y_k = {}_kQ_k, \tag{6} $$

$$ {}_lY_{ijk}Y_l = {}_lR_l \tag{7} $$

it is possible to extract from these matrices the component loadings matrices ${}_iA_m$, ${}_jB_p$, ${}_kC_q$ and ${}_lD_r$. Eigensolutions obtained from the cross-product matrices would yield the following series of component solutions:

$$ {}_iM_i \cong {}_iA_m S_m A_i, \tag{8} $$

$$ {}_jP_j \cong {}_jB_p U_p B_j, \tag{9} $$

$$ {}_kQ_k \cong {}_kC_q V_q C_k, \tag{10} $$

$$ {}_lR_l \cong {}_lD_r W_r D_l, \tag{11} $$

where ${}_mS_m$, ${}_pU_p$, ${}_qV_q$, and ${}_rW_r$ are diagonal matrices containing the nontrivial characteristic roots of the corresponding cross-product matrices and ${}_iA_m$, ${}_jB_p$, ${}_kC_q$, and ${}_lD_r$ are the respective loading matrices containing the unit-length nontrivial characteristic vectors.

As ${}_iA_m$, ${}_jB_p$, ${}_kC_q$, and ${}_lD_r$ are column-wise sections of orthonormal matrices, then the left-inverse of any of the loadings matrices are their transposes. Since ${}_mA_i A_m = {}_mI_m$, (2) can be rearranged to form:

$$ {}_mA_i Y_{jkl} \cong {}_mG_{pqr}[({}_pB_j \mathbf{X}_q C_k)\mathbf{X}_r D_l]. \tag{12} $$

As the Kronecker product of two column-wise sections of orthonormal matrices is itself a column-wise section of an orthonormal matrix [Bellman, 1960], then the products $({}_pB_j \mathbf{X}_q C_k)$ and $[({}_pB_j \mathbf{X}_q C_k)\mathbf{X}_r D_l]$ are column-wise sections of orthonormal matrices. Thus, the left-hand side of (12) times the transpose of $[({}_pB_j \mathbf{X}_q C_k)\mathbf{X}_r D_l]$ provides an estimate of the core matrix G. Since the transpose of a Kronecker product matrix is the Kronecker product of the transposes of the original matrices [Bellman, 1960], then [transpose $[({}_pB_j \mathbf{X}_q C_k)\mathbf{X}_r D_l]] = [({}_jB_p \mathbf{X}_k C_q)\mathbf{X}_l D_r]$. This allows (12) to be rewritten as

$$ {}_mA_i Y_{jkl}[({}_jB_p \mathbf{X}_k C_q)\mathbf{X}_l D_r] \cong {}_mG_{pqr}, \tag{13} $$

which provides a means for estimating core matrix values.

As in standard principal component analysis, it is possible to consider rotations or transformations of the four-mode loading matrices. These transformations can be represented as:

$$ {}_iA_{m^*} = {}_iA_m T_{m^*} \tag{14} $$

$$ {}_jB_{p^*} = {}_jB_p T_{p^*} \tag{15} $$

$$ {}_kC_{q^*} = {}_kC_q T_{q^*} \tag{16} $$

$$ {}_lD_{r^*} = {}_lD_r T_{r^*} \tag{17} $$

where ${}_iA_{m^*}$, ${}_jB_{p^*}$, ${}_kC_{q^*}$, and ${}_lD_{r^*}$ are the transformed loadings matrices and ${}_mT_{m^*}$, ${}_pT_{p^*}$, ${}_qT_{q^*}$, and ${}_rT_{r^*}$ are the square, nonsingular transformation matrices.

If rotations have been performed, then this requires a four-mode model taking into account not only the rotated loadings matrices, but also a core matrix whose basic modes reflect the transformations. Following the form of (2), the effect of the transformations is as follows:

$$ {}_iY_{jkl} \cong {}_i\hat{Y}_{jkl} = {}_iA_{m^*} G_{p^*q^*r^*}[({}_{p^*}B_j \mathbf{X}_{q^*} C_k)\mathbf{X}_{r^*} D_l] \tag{18} $$

where ${}_{m^*}G_{p^*q^*r^*}$ is the transformed core matrix defined as:

$$ {}_{m^*}G_{p^*q^*r^*} \cong ({}_mT_{m^*})^{-1}{}_mG_{pqr}(({}_{p^*}T_p)^{-1}\mathbf{X}({}_{q^*}T_q)^{-1})\mathbf{X}({}_{r^*}T_r)^{-1}. \tag{19} $$

Equation (19) is justified by, first, presenting the inverse of the transformations stated in (14)–(17):

$$ {}_iA_{m^*}({}_mT_{m^*})^{-1} = {}_iA_m \tag{20} $$

$$ {}_jB_{p^*}({}_pT_{p^*})^{-1} = {}_jB_p \tag{21} $$

$$ {}_kC_{q^*}({}_qT_{q^*})^{-1} = {}_kC_q \tag{22} $$

$$ {}_lD_{r^*}({}_rT_{r^*})^{-1} = {}_lD_r. \tag{23} $$

Second, substitution of (20)–(23) into (2) yields:

$$ {}_iY_{jkl} \cong {}_i\hat{Y}_{jkl} = {}_iA_{m^*}({}_mT_{m^*})^{-1}{}_mG_{pqr}[(({}_{p^*}T_p)^{-1}{}_{p^*}B_j \mathbf{X}\ ({}_{q^*}T_q)^{-1}{}_{q^*}C_k)\mathbf{X}({}_{r^*}T_r)^{-1}{}_{r^*}D_l]. \tag{24} $$

Third, since the Kronecker product of products equals the product of Kronecker products [Bellman, 1960], then (24) can be rewritten as:

$$ {}_iY_{jkl} \cong {}_i\hat{Y}_{jkl} = {}_iA_{m^*}[({}_mT_{m^*})^{-1}\,{}_mG_{pqr}(({}_{p^*}T_p)^{-1} \otimes ({}_{q^*}T_q)^{-1}) \otimes ({}_{r^*}T_r)^{-1}][({}_{p^*}B_j \otimes {}_{q^*}C_k) \otimes {}_{r^*}D_l]. \quad (25) $$

Therefore, the equivalence of (25) with (18), justifies (19).

If considering only orthonormal transformation matrices then the logic used to derive (13) from (2) can be applied to (18) to yield an alternative way of estimating the transformed core matrix:

$$ {}_{m^*}A_i\,Y_{jkl}[({}_jB_{p^*} \otimes {}_kC_{q^*}) \otimes {}_lD_{r^*}] \cong {}_{m^*}G_{p^*q^*r^*}\,. \quad (26) $$

Despite the similarity to ordinary principal components analysis, the computational methods presented for the four-mode model do not provide a least-squares fit to a four-mode data matrix. These computational procedures, however, *may* provide solutions that are adequate in practice. Since the potential for poor fits exists, especially as the sum of deleted roots (over modes) increases, application of the four-mode model should include assessment of the correspondence between ${}_i\hat{Y}_{jkl}$ and ${}_iY_{jkl}$.

Applying Four-Mode Component Analysis

To illustrate the four-mode model as more than an exercise in matrix algebra, the model's application to some real data will be examined. The data analyzed is a four-mode matrix consisting of a set of individuals' responses to repetitive advertising stimuli on a battery of Likert-type items. More specifically, 27 individuals were exposed to a set of 6 different television advertisements, on 5 separate exposure occasions over a month's time. After each exposure to every ad the individuals responded to a battery of 16 different items designed to measure viewer reaction to television advertising. The items, all of which used a 1–6 "Strongly Disagree–Strongly Agree" scale, were selected from a larger battery of items known as the Viewer Response Profile [Leo Burnett Company, Note 1; Schlinger, 1979].

For the current analysis the i, j, k and l modes of the data corresponded to individuals, exposure occasions, advertisements, and items, respectively. Consequently, the four-mode data matrix X contained the elements $x_{i'j'k'l'}$ such that $i' = 1, 2, 3, \ldots, 27$; $j' = 1, 2, 3, 4, 5$; $k' = 1, 2, 3, 4, 5, 6$; and $l' = 1, 2, 3, \ldots, 16$. The data were transformed to represent standard scores for each of the 16 items scaled by the inverse of the square root of the sample size. That is, first, the data matrix ${}_{ijk}X_l$ was standardized such that each column summed to zero and had a variance of one. Second, the data was multiplied by $1/(27)^{1/2}$. This allowed the cross-products matrix in (7), ${}_lR_l$, to take the form of a correlation matrix. The effect of such transformations is to make the data matrix Y represent values directly proportional to standardized values on each item, when pooling over individuals, ads and exposures.

The number of components retained in each mode was determined with several criteria. Theory suggesting the number and types of cognitive responses viewers experience in reaction to repetitive television advertising [Krugman, 1972, 1975], the interpretability of the rotated component solutions, and the relative size of the characteristic roots extracted from the cross-product matrices all played a role in deciding on the number of components. As the underlined characteristic roots in Table 1 reveals, these three considerations suggested: $m = 4$, corresponding to four idealized individuals; $p = 3$, corresponding to three psychological exposures; $q = 2$, corresponding to two basic types of ads and $r = 3$, corresponding to three latent scales in the items. The rank of the exposure and item modes, three, is consistent with theoretical expectations.

TABLE 1

Characteristic Roots of Product Matrices†

Root Number	Product Matrices			
	ℓ Mode (Items) $_{\ell}R_{\ell}$	k Mode (Ads) $_{k}Q_{k}$	j Mode (Exposures) $_{j}P_{j}$	i Mode (Individuals) $_{i}M_{i}$
1	5.954	7.344	7.875	3.105
2	2.271	2.644	4.153	1.863
3	1.282	2.061	1.735	1.177
4	1.208	1.555	1.172	1.123
5	.972	1.260	1.064	.909
6	.787	1.134		.773
7	.578			.676
8	.538			.574
9	.474			.519
10	.449			.472
11	.365			.448
12	.330			.422
13	.268			.415
14	.252			.399
15	.166			.365
16	.130			.342
17				.311
18				.284
19				.273
20				.263
21				.251
22				.224
23				.202
24				.182
25				.172
26				.143
27				.106

†Smallest non-trivial root retained in each mode is underlined.

Varimax rotated component loadings matrices for the item, exposure, and advertisement modes are shown in Tables 2, 3 and 4. Since the individual mode loadings matrix would add little to understanding the current analysis, it is not presented. The individual mode loadings matrix, like the other loadings matrices, was also varimax rotated. The exclusive use of varimax rotations in this analysis should not convey that only orthonormal rotations are appropriate with the four-mode model. A transformation matrix is appropriate as long as it is nonsingular and yields a meaningful transformed loadings matrix.

The rotated item components, or scales, shown in Table 2 are similar to major dimensions regularly found in advertising response data [e.g., Schlinger, 1979; Wells, Leavitt & McConville, 1971; Leavitt, 1970]. The first item component reflects the degree viewers claim to perceive an advertisement and its advocated consumer product as relevant to their own needs. Component I_{r*}, therefore, may be interpreted as a scale measuring "Personal Evaluation." The second item component seems to measure the degree viewers claim to understand a television commercial's message. Thus, II_{r*} was labeled "Comprehension." The third latent scale appears to characterize reactions to

TABLE 2

Varimax Rotated ℓ Mode Component Loadings Matrix:[†] $_{\ell}D_{r*}$

Item Number	Item	I_{r*}	II_{r*}	III_{r*}
2	During the commercial I thought how the product might be useful for me.	.435	-.037	-.049
3	I felt as though I was right there in the commercial experiencing the same thing.	.314	-.038	-.140
6	The commercial was meaningful to me.	.446	.043	.040
8	The ad did not have anything to do with me or my needs.	-.439	-.055	-.210
11	The commercial gave me a good idea.	.306	-.006	-.126
14	As I watched I thought of reasons why I would buy or not buy the product.	.418	-.044	.042
5	I clearly understood the commercial.	.136	.366	.072
7	The commercial was too complex. I was not sure what was going on.	-.001	-.536	-.011
12	I was not sure what was going on in the commercial.	-.009	-.531	-.006
15	I was so busy watching the screen, I did not listen to the talk.	.083	-.339	.024
16	The commercial went by so quickly that it just did not make an impression on me.	.070	-.405	.074
1	The commercial was lots of fun to watch and listen to.	.076	.003	-.438
4	I have seen this commercial before.	.095	.056	.372
9	I have seen this commercial so many times that I am tired of it.	.013	-.050	.434
10	I thought the commercial was clever and quite entertaining.	.050	.036	-.428
13	The ad was not just selling--it was entertaining me. I appreciated that.	.051	-.002	-.448

[†]Salient loadings used in interpretation are underlined.

advertising exposure as either enjoyable or irritating. This component, III_{r*}, seems to tap "Emotive Response," reflecting overall positive or negative viewer reaction.

Table 3 contains the rotated exposure mode component matrix. The loading pattern suggests the responses on the 16 Likert-type items during the first exposure to each ad were different from responses to subsequent exposures to the ads. The loadings also suggest the second and third exposures were much alike, but at the same time different from other exposures. Similarly, the fourth and fifth exposures were alike, but different

TABLE 3

Varimax Rotated j Mode Component Loadings Matrix:† ${}_j B_{p*}$

Number of Advertising Exposures	I_{p*}	II_{p*}	III_{p*}
1	.991	-.016	.027
2	.076	.842	-.195
3	-.095	.534	.271
4	.036	.072	.639
5	-.013	-.036	.692

†Salient loadings used in interpretation are underlined.

from other exposures. Such a loading pattern is consistent with Krugman's [1972, 1975] "Three-Exposure Theory" which argues that although viewers may see an ad dozens of times, psychologically, the viewers only experience three types of exposures. According to this theory the first psychological exposure corresponds to the first actual exposure. The second and third or more real exposures corresponds to a second psychological exposure. After a viewer has experienced the second psychological exposure, all subsequent real exposures whether they be the fourth, fifth and sixth or twenty-second and twenty-third are repeats of the third psychological exposure. Therefore, it seems appropriate to interpret I_{p*}, II_{p*}, and III_{p*} as Krugman's first, second and third psychological exposures, respectively. This loading pattern, however, provides only tentative support for Krugman's three psychological exposures. Since data on only up to five exposures were collected, it is unknown if additional data collected for more than five exposures would load on the third component and not require additional components.

TABLE 4

Varimax Rotated k Mode Component Loadings Matrix:† ${}_k C_{q*}$

Product Advertised	Commercial Name	I_{q*}	II_{q*}
United Airlines	"Big Day"	.337	.145
United Airlines	"Mother Country"	.594	-.164
Audi Fox	"Duke of Klaxton"	.467	.096
Audi Fox	"Fox Hunt"	.560	-.045
Allstate Auto Insurance	"Fender Bender"	.015	.681
Allstate Auto Insurance	"Chicago"	.027	.691

†Salient loadings used in interpretation are underlined.

The rotated loadings matrix for the advertisement mode in Table 4 suggests two advertisement types. Component I_{q^*} appears to represent a group of four different ads for two different products, an airline and an imported automobile; while II_{q^*} is a pair of ads for a single product, an automobile insurance. Besides the consumer product being promoted, the two sets of ads also differed in other important ways. For instance, in comparison to the insurance ads, the airline and automobile ads contained a more complex message and made more use of drama, comedy and music in their creative treatments.

One-to-one component to idealized entity interpretations, as offered in the exposure and advertisement component mode loading discussions, may not always be appropriate with analyses of other data sets. Since the number of meaningful points or idealized entities in a component space can be greater than the number of components, the expectation of a rigid isomorphic component to type or idealized entity relationship is misleading. Such one-to-one interpretations are only reasonable with analytic results as shown in Tables 2, 3 and 4 where each element in a mode from the original data loads on only one latent component.

Corresponding to the number of components retained for the individual, exposure occasion, advertisement and item modes, the core matrix for this analysis is of order $4 \times 3 \times 2 \times 3$. The core matrix, transformed to reflect varimax rotations on all four modes, is shown in Table 5. The four-dimensional matrix when written as ${}_{m^*}G_{p^*q^*r^*}$, can be thought of containing scores of the four idealized viewer types on pairs of components representing combinations of different types of ads, psychological exposures and response scales. That is, the core matrix elements indicate the reactions (on three scales) of four different types of idealized viewers to two different groups of ads at three fundamental repetitive exposure levels.

The comprehension scores in the second and fifth columns of the core matrix in Table 5 suggest that with increased repetition, the respondents generally claimed to know relatively more about the contents of the ads. This pattern of increased comprehension with additional exposures is consistent with Krugman's theory. The comprehension scores also show differences in respondent types. Types I and II claimed to understand the ads more than Types III and IV.

Inspection of the personal relevance scores in the first and fourth columns of ${}_{m^*p^*}G_{q^*r^*}$ in Table 5 shows an almost universal phenomenon. In almost every pair of respondent and advertisement type, viewer questioning of advertisement message content relevance was most intense during the second exposure. Although this is inconsistent with cognitive theories of perception which claim information is first evaluated in terms of its pertinence, this is consistent with the Krugman "Three Exposure Theory." Krugman argues that, while message content is decoded during the first psychological exposure, evaluation of message relevance primarily occurs during the second psychological exposure. Differences between Respondent Types I and II versus III and IV are also apparent on personal evaluation. Across ad types, Respondent Types III and IV viewed the advertisements as having more personal relevance then Respondent Types I and II. When viewed simultaneously, columns one, two, four and five of ${}_{m^*p^*}G_{q^*r^*}$ suggest that viewers think they know a good deal about the content of an irrelevant ad after a few exposures, while when viewers see the ads as relevant the ads are viewed as not having precise or clear enough information.

The emotive response scores in the core matrix reveal individual differences in perceptions of the two different types of advertisements. For instance, whereas Respondent Type I found the airline and automobile ads more enjoyable with increased repetition and grew more annoyed with additional exposures to the insurance ads; Respondent Type II's emotive reactions to the ad types across exposures showed a reverse trend.

TABLE 5

Transformed Core Matrix:† $_{m^*p^*}G_{q^*r^*}$

		Airline and Automobile Ads: I_{q^*}			Insurance Ads: II_{q^*}		
		Personal Evaluation:†† I_{r^*}	Comprehension:§ II_{r^*}	Emotive Response:¶ III_{r^*}	Personal Evaluation:†† I_{r^*}	Comprehension:§ II_{r^*}	Emotive Response:¶ III_{r^*}
Idealized Respondent One: I_{m^*}	1st Psychological Exposure: I_{p^*}	-.156	.120	.096	-.058	-.002	.004
	2nd Psychological Exposure: II_{p^*}	-.100	.004	.075	-.011	.098	.070
	3rd Psychological Exposure: III_{p^*}	-.187	.120	.053	-.187	.146	.083
Idealized Respondent Two: II_{m^*}	1st Psychological Exposure: I_{p^*}	-.217	-.044	.018	-.274	.084	.187
	2nd Psychological Exposure: II_{p^*}	.460	-.270	.080	.594	.198	-.069
	3rd Psychological Exposure: III_{p^*}	-1.438	.845	.405	-.784	.263	.135
Idealized Respondent Three: III_{m^*}	1st Psychological Exposure: I_{p^*}	.314	-.378	-.221	.107	-.146	.027
	2nd Psychological Exposure: II_{p^*}	.620	-.242	-.176	.390	-.201	-.228
	3rd Psychological Exposure: III_{p^*}	-.002	-.042	-.003	.089	-.319	-.140
Idealized Respondent Four: IV_{m^*}	1st Psychological Exposure: I_{p^*}	.277	-.206	.114	.194	-.161	-.081
	2nd Psychological Exposure: II_{p^*}	.305	-.187	-.149	.156	-.203	-.104
	3rd Psychological Exposure: III_{p^*}	.510	-.347	-.170	.316	-.257	-.121

† Core matrix values presented have been multiplied by ten.

†† Higher numerical values on the "Personal Evaluation" scale represents relatively higher perceived relevance.

§ Higher numerical values on the "Comprehension" scale represents relatively higher perceived understanding.

¶ Higher numerical values on the "Emotive Response" scale indicates relatively higher irritation with the ad as a whole.

At its face, this four-mode model appears to characterize the repetitive Viewer Response Profile data. The mode is interpretable and it conforms, in part, with some theoretical expectations about how repetitive advertising works. The computed product-moment correlation between the $\hat{y}_{i'j'k'l'}$'s and the $y_{i'j'k'l'}$'s, .88, further confirms the adequacy of the model's representation of the data.

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Manuscript received 4/24/80
Final version received 10/29/80