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THREE-MODE FACTOR ANALYSIS[1]

JOSEPH LEVIN

University of Illinois[2]

2 studies employing Tucker's 3-mode factor analysis are reported. The 1st is an analysis of semantic differential data. 4 scale factors and 4 concept factors were obtained. 1 subject-type was obtained. It consisted of a core matrix linking the scale factors to the concept factors. The 2nd study is an analysis of an S-R Inventory of Anxiousness. 3 situation factors and 3 response factors were obtained. Also, 3 types were obtained, each consisting of a 3 × 3 matrix linking the response factors to the situation factors.

Classical factor analytic procedures are applicable to two-way classification data; for example, to matrices of subjects by tests. Assuming that the data have already been standardized properly, the data matrix can be factored into a product of two matrices: a factor score matrix and a factor loading matrix. However, psychological data are often three-way (or multi-way) classification data; for example, multi-trait multi-method matrices (Campbell & Fiske, 1959), the semantic differential (Osgood, Suci, & Tannenbaum, 1957), and the S-R Inventory of Anxiousness (Endler, Hunt, & Rosenstein, 1962).

The problem of analyzing multi-mode data (i.e., multi-way classification tables) has been discussed in the literature frequently (Abelson, 1960; Burt, 1955; Cattel, 1952; Guttman, 1958; Mahmoud, 1955). Yet all the proposed solutions are based on some sort of reduction of the data to a two-way table. The semantic differential is frequently reduced to a concept by scale table, averaging over subjects. Whenever individual differences are considered important, and the mode of "subjects" cannot be eliminated, the items are summed (or averaged) over the "unimportant" mode; for example, data given in a table of subjects by tests by occasions can be reduced by averaging over occasions. The S-R Inventory of Anxiousness (Endler, Hunt, & Rosenstein, 1962), which consists of responses to situations by subjects, can be reduced to a response by subject table, averaging each response over all situations. The latter is an example where the asymmetrical treatment, involved in most reduction procedures, is not tenable. Indeed, in their analysis of the S-R Inventory, Endler et al. have perceived this point and have treated the data symmetrically. Summing situations, they obtained a subject by response matrix. Summing responses, they obtained a subject by situation matrix. They also pointed out the problem of the relation among the factors obtained from each of these matrices, respectively.

The problem of a simultaneous analysis of all the modes of the data and of the relation among the corresponding factors is the main issue of three-mode factor analysis.

The Logic of Three-Mode Factor Analysis

The mathematical problem of three-mode factor analysis has been solved by L. R Tucker[3] (Tucker, 1964). The following section is an exposition of the structure and logical meaning of this method.

Three-mode factor analysis is not a straightforward generalization of classical two-mode factor analysis. Two-mode factor analysis requires some modification before it can be generalized. It is, therefore, simpler, first to

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[2] Now at the Szold Foundation and the University of Tel-Aviv, Israel.

[3] Implication of factor analysis of three-way matrices for measurement of change. Paper presented at the Conference in Measuring Change, University of Wisconsin, May 1962.

introduce a modified version of two-mode factor analysis and explain three-mode factor analysis as a straightforward generalization. Besides, this modfication also has intrinsic interest in the two-mode case.

A Modification of Two-Mode Factor Analysis

Let us reduce the semantic differential data (subjects by scales by concepts) to a two-way classification averaging over subjects. We obtain a scales by concepts matrix, X, of order $i \times k$ and of rank $p (p \leq i,\ p \leq k)$. Let us assume that the scores have already been standardized with regard to origin and unit of measurement, so that sum of products can be used instead of correlations. With these assumptions, the Eckart-Young procedure can be employed (Eckart & Young, 1936). According to this procedure ${}_iX_k$ can be factored as a product of three matrices:

$$ {}_iX_k = {}_iU_i \Lambda_k V_k, \qquad [1] $$

where U = the orthonormal latent vector matrix of XX^T,
V = the orthonormal latent vector matrix of X^TX,
Λ = a rectangular matrix with 0 entries off diagonally, p nonzero entries ($\lambda_1 \cdots \lambda_p$) in the diagonal starting with the upper left corner, and λ_j^2 ($j = 1 \cdots p$) are the latent roots of XX^T (or of X^TX). Without loss of generality it may be assumed that the λ_j are arranged in decreasing order of magnitude.

If the m largest values of Λ are chosen, then

$$ {}_i\ddot{X}_k = {}_iU_m \Lambda_m V_k \qquad [2] $$

is a least square approximation to X for m factors. This can be reformulated so that U and V become principal-axes factor loading matrices:

$$ \begin{aligned} \ddot{X} &= (U\Lambda)\Lambda^{-1}(\Lambda V) \\ &= \bar{U}\Lambda^{-1}\bar{V}; \end{aligned} \qquad [3] $$

$\bar{U}$ and $\bar{V}$ are the factor loading matrices for scales and concepts, respectively. The next step is to introduce a rotation to simple structure. Due to the logical symmetry of concepts and scales we have to rotate both $\bar{U}$ and $\bar{V}$.

Let $\bar{U}$ be rotated by T, and $\bar{V}$ be rotated by P

$$ \ddot{X} = (\bar{U}T)T^{-1}\Lambda^{-1}P^{-1}(PV). \qquad [4] $$

Defining

$$ A = \bar{U}T, \quad B = P\bar{V}, \quad G = T^{-1}\Lambda^{-1}P^{-1}, \qquad [5] $$

we obtain

$$ \ddot{X} = AGB. \qquad [6] $$

The factor analytic solution is now a product of three matrices instead of the usual solution, $X = AB$. This seems to be a loss of parsimony, but this loss is compensated by a gain: both scales and concepts are treated symmetrically. We have a factor loading matrix for scales and a factor loading matrix for concepts. And both matrices have been transformed to some especially meaningful form such as simple structure.

In the formula AGB the two matrices A and B have a symmetrical role and both are analogous to factor loadings, that is, they can be explained as weights applied to factor scores. Therefore, G functions as a factor score matrix. But, unlike the usual factor analytic formula for the factor scores, the matrix G is reduced with respect to both rows and columns. To return to the semantic differential example: both scales and concepts are reduced to factors, and the internal core matrix G gives the factor scores of concept factors on scale factors.

A test by subjects paradigm may clarify the issue in more familiar terms, though the requirement for symmetry is not as cogent here. It is also convenient to use in this context the terminology "idealized entities," like "idealized subjects," and "idealized traits," (Helm & Tucker, 1962, p. 439; Tucker & Messick, 1960, p. 5).

In the fundamental equation of classical factor analysis,

$$ {}_iX_k = {}_iA_m B_k, \qquad [7] $$

${}_iX_k$ is a matrix of subjects by tests, ${}_iA_m$ is a matrix of subjects by factors, and ${}_mB_k$ is a matrix of factors by tests. In the matrix ${}_mB_k$, each row can be interpreted as the entries of an "idealized subject." Employing this terminology, each idealized subject is a k-tuple consisting of the loadings of the k tests on a factor.

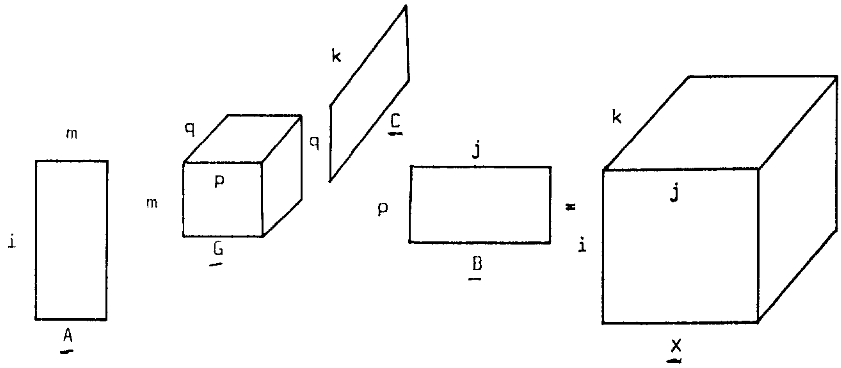

Fig. 1

Sectioning matrix B by rows we obtain:

$$\begin{aligned}(x_{j1}, x_{j2} \cdots x_{jk}) &= a_{j1}\,(b_{11}, b_{12} \cdots b_{1k}) \\ &+ a_{j2}(b_{21}, b_{22} \cdots b_{2k}) \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &+ a_{jm}(b_{m1}, b_{m2} \cdots b_{mk}).\end{aligned} \qquad [8]$$

The row $(x)_j$ is a vector of predicted scores for a given subject j on the k tests. This row-vector is a weighted sum of the rows of B. The rows of B are the scores of the set of idealized subjects, and the row vector $(a)_j$ is a row of weights for subject j.

Now we have to modify this interpretation in order to adapt it to two-mode factor analysis with an inner core matrix. In the formula

$$_iX_k = {}_iA_m G_m B_k \qquad [9]$$

the roles of subjects and tests are symmetric. $_mG_m$ is a matrix of scores of idealized subjects on idealized tests. The rows of G are for idealized subjects and the columns are for idealized tests. The A and B matrices are weights. Each row of the matrix $_iA_m$ is a set of weights for a given subject. Applying these weights to the idealized subjects (i.e., to the rows of G) we obtain the scores of the subject expressed as a linear combination of the idealized subjects. Similarly, each column of $_mB_k$ is a set of weights for a given test. Applying these weights to the idealized tests (i.e., the columns of G) we obtain the scores on the test expressed as a linear combination of the idealized tests. Obviously, the pre- and postmultiplication of G by A and B produces the scores (or predicted scores if nonsignificant factors are omitted) of all subjects on all tests from the scores of the idealized subjects on the idealized tests.

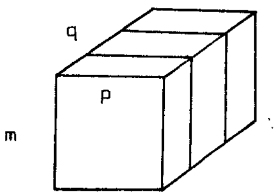

Fig. 2

An Extension to Three-Mode Factor Analysis

We can extend this formulation to three-mode factor analysis. The fundamental formula of three-mode factor analysis is:

$$x_{ijk} = \sum_m \sum_p \sum_q a_{im} b_{jp} c_{kq} g_{mpq}. \qquad [10]$$

The three-mode factor analysis starts with data given in a three-way classification. These data are reduced to an inner core G_{mpq}, which is again of three-way classification, and three factor weight matrices, $_iA_m$, $_jB_p$, $_kC_q$. The inner core, G, is analogous to the inner core in the equation $X = AGB$. The A, B, and C matrices are analogous to the factor loading matrices. These relations are described diagrammatically in terms of boxes and rectangles in Figure 1. The box G can be sectioned into q frontal matrices of order $m \times p$ each, as in Figure 2. Each of these is premultiplied by A and postmultiplied by B. The resulting box can now be sectioned laterally into j matrices of order $i \times q$ each. Each of these can now be postmultiplied by C as shown in Figure 3.

These operations reproduce the box X of order $i \times j \times k$. The multiplication of G_{mpq} by the three linear operators A, B, and C, can be performed in any order; that is, we can start with multiplication of G by A, then by B, and finally by C. We can also start with C, then multiply by B, and finally by A, and so on. This follows from the commutativity of multiplication and of summation in the fundamental formula, 10.

Now the generalization of two-mode factor analysis becomes obvious. Let us return to the example of the semantic differential. The data are given as concepts by scales by subjects (raters). We obtain three weight matrices (or factor loading matrices): one for concepts, one for scales, and one for subjects. This establishes three sets of factors. Then, we get an inner core "box." In analogy to the two-mode case, this gives the interrelations that connect the three sets of factors, that is, the factor score of each concept factor on each scale factor for each subject factor.

Employing the terminology of idealized entities, the inner core box gives the scores of each idealized concept on each idealized scale for each idealized subject; and the three outer matrices are weights applied to these idealized entities needed to reproduce the original scores of subject per scale per concept.

The Semantic Differential

Data

This study is a reanalysis of semantic differential data collected by Edward Ware (1958). There are 31 concepts rated on 20 scales by 60 subjects. The subjects are high-school students sampled according to the scheme indicated in Table 1.

The scales had seven categories (1–7) and were presented in graphical format.

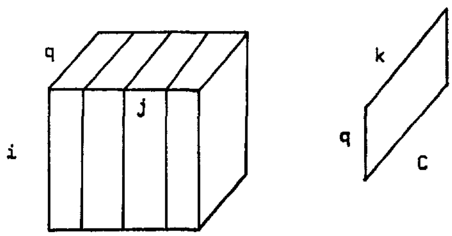

Fig. 3

TABLE 1

Distribution of Subjects in the Semantic Differential Study by Sex and IQ

	Male	Female	Totals
High IQ	15	15	30
Low IQ	15	15	30
Total	30	30	60

Method

Scoring. The data were rescored from a 1 to 7 scale to a +3 to −3 scale. Assuming that 0 (the scale midpoint) is a natural neutrality point, and assuming that differences in size of unit (or standard deviation) reflect individual differences that ought to be retained in the analysis, no additional standardization seems to be required. The scores were, therefore, treated as standard scores, and sums of products were employed throughout, instead of correlations.

There are two ways to convert the data to standard scores that can be suggested as intuitively meaningful. One method is standardization of each scale as a variable, pooled over concepts and subjects. The other one is a separate standardization of each subject, which is essentially a conversion to ipsative scores. The effects of the mean and the unit of measurement have to be discussed separately.

The mathematical formulas expressing the effects of elimination of the mean (or any additive constant) are quite cumbersome. Fortunately, the deviations of the mean of each scale from zero were negligible so that this issue did not have practical significance.

On the other hand, the effect of the unit of measurement is quite simple. It follows from Formula 10 that a multiplicative constant can be incorporated in any of the matrices A, B, C depending on its subscripts (i.e., on the method of standardization). As shown in the next section, the score matrices of each subject are proportional; and by a conversion of each subject to a standard scale, we would lose some interesting information.

Factor Loading Matrices. Let i be the subscript for subjects, j for concepts, k for scales; i', j', and k' are used as alternative subscripts to i, j, and k. Three matrices of sums of products were computed:

$$\sum_j \sum_k x_{ijk} x_{i'jk}\,, \qquad [11]$$

$$\sum_i \sum_k x_{ijk} x_{ij'k}\,, \qquad [12]$$

$$\sum_i \sum_j x_{ijk} x_{ijk'}\,. \qquad [13]$$

These matrices were analysed by the method of principal axes and rotated to simple structure. The resulting factor loading matrices are the matrices A, B, and C of Equation 10. The inner core is obtained as a least square solution[4] (Levin, 1963).

[4] L. R Tucker, Analysis procedures in three-mode factor analysis, in preparation.

TABLE 2

FOUR SCALE FACTORS—VARIMAX ROTATION

Scales	Factor I	Factor II	Factor III	Factor IV
1. Fast-Slow	−05	58[a]	10	15
2. Even-Uneven	06	06	67[a]	03
3. Strong-Weak	22	66[a]	00	−02
4. Good-Bad	61[a]	−01	46[a]	−08
5. New-Old	06	−12	24	10
6. Warm-Cool	05	02	04	54[a]
7. Moral-Immoral	52[a]	11	11	−01
8. True-False	56[a]	20	−03	−06
9. Predictable-Unpredictable	02	−05	61[a]	−16
10. Sociable-Unsociable	40[a]	12	43[a]	09
11. Hot-Cold	−06	04	03	52[a]
12. Hard-Soft	−41[a]	46[a]	−13	−26[a]
13. Fair-Unfair	60[a]	−09	−15	03
14. Usual-Unusual	66[a]	05	−13	−04
15. Kind-Cruel	56[a]	−17	22	07
16. Straight-Twisted	16	−05	28[a]	04
17. Powerful-Powerless	24	55[a]	−23	08
18. Colorful-Colorless	33[a]	09	19	19
19. Simple-Complex	16	−25	05	−23
20. Emotional-Unemotional	14	28[a]	−32[a]	38[a]

[a] Salient loadings.

Results

Number of Factors. The number of factors has been determined, primarily, on basis of the latent roots. If we plot the latent roots in decreasing order of magnitude we find that the roots drop abruptly at the beginning, then taper off slowly. The bending point is used as a cutting point to determine the number of factors.

There is, apparently, only one major factor for subjects. The latent roots drop abruptly after the first root: 45,900; 5,500; 4,460; 3,650; 3,470; 3,270; 3,110; 2,290; etc. Correspondingly, only one idealized subject is retained in the inner core. Consequently, for these data, the predicted scores of each subject are computed by multiplication of the idealized subject by a constant of proportionality. Employing a somewhat different procedure, Edward E. Ware arrived at the same conclusion.[5]

Four factors have been retained for concepts. The latent roots are 26,890; 22,520; 9,470; 7,440; 4,810; 4,340; 3,870; 3,800; etc. From the fourth root onward the roots taper off gradually and slowly.

[5] Personal communication.

The results for scales are not so clear-cut. The latent roots are 31,090; 18,140; 10,910; 8,920; 7,670; 6,510; 5,990; 5,760; etc.

There are either four or five factors. Considerations of meaningful interpretation support the decision to retain four factors for scales. The pattern obtained from a rotation of four scale factors provides a meaningful interpretation, and the factors can be identified in terms of factors commonly obtained in semantic differential studies. The pattern obtained by rotation of five scale factors was ambiguous.

Corresponding to the number of factors

TABLE 3

FOUR CONCEPT FACTORS—VARIMAX ROTATION

Concepts	Factor I	Factor II	Factor III	Factor IV
1. Food	38[a]	−07	12	13
2. Baby	52[a]	−08	15	−20
3. Pig	07	21	14	04
4. Grief	01	43[a]	00	−02
5. Silk	21	−12	30[a]	09
6. Snow	21	02	43[a]	01
7. Bottom	−02	12	14	18
8. Sickness	−02	50[a]	04	−12
9. Mother	64[a]	−14	03	17
10. Army	20	15	−05	45[a]
11. Job	08	22	21	−23
12. Jelly	15	−10	43[a]	20
13. Street	14	10	17	35[a]
14. Fear	03	53[a]	01	13
15. Anger	04	54[a]	−15	14
16. Insane man	−08	50[a]	−06	−04
17. Me	45[a]	−05	00	15
18. Cop	22	13	−04	36[a]
19. Sin	−10	55[a]	−09	05
20. Sleep	40[a]	−07	25	15
21. Pain	04	49[a]	−06	09
22. Spider	−07	33[a]	16	05
23. Butter	11	−12	44[a]	10
24. Lamp	23	−06	15	33[a]
25. Dream	34[a]	15	12	02
26. Chair	18	−13	28	25
27. Snail	02	02	39[a]	06
28. Sex	50[a]	15	01	13
29. Dirt	−02	14	36[a]	10
30. Statue	00	−04	22	39[a]
31. Beggar	−04	08	08	00

[a] Salient loadings.

retained for subjects, concepts, and scales, the inner core is of order $1 \times 4 \times 4$.

Factor Loading Matrices and Interpretation of Factors. The factor matrices are given in Tables 2 and 3. Only the results of Varimax rotation (Harman, 1960, ch. 14) are reported here. The results of an oblique rotation were essentially the same.

The factors can be identified on the basis of salient variables. The scale factors are essentially similar to the major factors regularly obtained in semantic differential studies.

The first factor has high loadings on the following scales:

14. Usual-Unusual	66
4. Good-Bad	61
13. Fair-Unfair	60
8. True-False	56
15. Kind-Cruel	56
7. Moral-Immoral	52

This is the Evaluation factor.

The second factor has high loadings on the following scales:

3. Strong-Weak	66
1. Fast-Slow	58
17. Powerful-Powerless	55
12. Hard-Soft	46

This is a coalescence of the Potency and Activity factors. It was also obtained in a study by Triandis and Osgood (1958) and was called a Dynamism factor.

The third factor has high loadings on these scales:

2. Even-Uneven	67
9. Predictable-Unpredictable	61
4. Good-Bad	46
10. Sociable-Unsociable	43
20. Emotional-Unemotional	−32

and possibly,

16. Straight-Twisted	28

This factor seems to come closest to a Stability factor.

The fourth factor has few high loadings. It is high on these scales:

6. Warm-Cool	54
11. Hot-Cold	52
20. Emotional-Unemotional	38

Scales 6, 11, and 20 are virtually synonymous, especially when one considers their frequent metaphorical use. This factor may be called a Warmth factor. While the identification of this factor seems logically reasonable on basis of the data obtained here, it has not appeared as a separate factor in other studies of the semantic differential.

Since factor analytic studies of the semantic differential report scale factors only, the identification and interpretation of concept factors is based entirely on the results obtained in this study.

The first factor has high loadings on the following concepts:

9. Mother	64
2. Baby	52
28. Sex	50
17. Me	45

and possibly,

20. Sleep	40
1. Food	38
25. Dream	34

These concepts are apparently related to family and home surroundings and may be called a Human factor or perhaps an Intimacy factor.

The second factor is high on these concepts:

19. Sin	55
15. Anger	54
14. Fear	53
8. Sickness	50
16. Insane man	50
21. Pain	49
4. Grief	43

These concepts signify intense emotional states related to agitated behavior. This may be called an Agitation factor.

The third factor has high loadings on the following concepts:

23. Butter	44
12. Jelly	43
6. Snow	43
27. Snail	39
29. Dirt	36

and possibly,

5. Silk	30

TABLE 4

Core Matrix Corresponding to the Varimax Rotation

Concept factors	Scale Factors: Evaluation I	Dynamism II	Stability III	Warmth IV
Intimacy I	18	03	−01	18
Agitation II	−03	10	−19	03
Slickness III	12	−13	−06	−15
Toughness IV	03	21	09	−14

These concepts signify soft, compressible and slick matter of shapeless and malleable material. This factor may be called a Slickness factor.

The fourth factor has high loadings on these concepts:

10. Army	45
30. Statue	39
18. Cop	36
13. Street	35
24. Lamp	33

and possibly,

2. Baby	−20

The highest loadings on the fourth factor are somewhat lower than those of the three preceding factors. It seems that they have in common a property of Toughness, whether tactile, such as in Statue, or metaphorical, such as in Army. Note also the negative loading of Baby.

The Core Matrix. The core matrix is given in Table 4.

In order to facilitate interpretation, a categorization based on rank order is adopted here, namely, the lowest five entries, the highest five entries, and the middle six entries. In Table 5 this corresponds to the following cutting points:

High:	|15|	and above
Medium:	between |03|	and |15|
Low:	|03|	and below

The core matrices can be represented schematically employing the following notation:

++	for high positive
+	for medium positive
0	for low or neutral
−	for medium negative
−−	for high negative

The schematic representation is given in Table 5.

TABLE 5

Core Matrix Corresponding to Varimax Rotation (Schematic Representation)

Concept Factors	Scale Factors: Evaluation I	Dynamism II	Stability III	Warmth IV
Intimacy I	++	0	0	++
Agitation II	0	+	−−	0
Slickness III	+	−	−	−−
Toughness IV	0	++	+	−−

The scores of the Agitation factor require a few comments. A similar relation between the concept "Mental disorder" and stability scales is reported by Nunally (1961). On the other hand, the neutral (or rather low negative) score of Agitation on the Evaluation factor defies explanation. An attempt to employ Oblimax rotation produced a shift downwards but not large enough to pull it out of the neutrality range.

The other entries of the table need no further comment.

The S-R Inventory of Anxiousness

Data

This study is a reanalysis of the data employed by Endler, Hunt, and Rosenstein (1962). This inventory consists of 14 responses to 11 situations. Each response is matched with each situation. The total number of items is 154.

> The choice of the situations . . . is based on an intuitive attempt to select a variety of situations that would be familiar through either direct or vicarious experience to most college freshmen or sophomores (the *S*s), a variety which would vary from the typically innocuous to the quite threatening.
>
> The S-R inventory employs a five-step scale, ranging from *none* to *very much*, with which *S* is asked to register the felt intensity of his own response to the situation in question [Endler et al., 1962, p. 5].

Here is an example of an item:

You are about to go on a roller coaster.
Heart beats faster 1 2 3 4 5
Not at all Much faster

The subjects were 169 male and female students in elementary courses in psychology at Pennsylvania State University.

Standardization of Scores. Unlike the semantic differential, it would be untenable to assume here that the midpoint of the scale of each item corresponds to the origin, neither can we assume that the units of measurement are equal.

The scores were converted to standard scores by the usual procedure of subtraction of the mean and division by the standard deviation. Note that there are $11 \times 14 = 154$ variables and there are 169 subjects for each variable.

Computation of Factor Loadings and Core. Though based on the same mathematical principles, a different method was employed here than with the semantic differential. The procedure provides another aspect of comparison between three-mode factor analysis and some classical procedures (Mahmoud, 1955).

Three correlation matrices were computed: (*a*) The intercorrelation of all items. This is a matrix of order 154. (*b*) The average intercorrelation of the situations. This is a matrix of order 11. (*c*) The average intercorrelation of the responses. This is a matrix of order 14. Each of these matrices was factor analyzed by by the method of principal axes.

The principal axes of the large matrix (154×154) were employed to compute the inner core.

The analysis of the 154 items (11 situations by 14 responses) yielded three significant factors. At this point, it seems that these factors generate a confounding of two logically distinct modes; for the loadings of each item involve both a situation and a response. However, rearranging the loadings of each factor in the format of a matrix of 11 situations by 14 responses, we obtain again a three-way classification table consisting of factors by situations by responses, thus keeping the distinct modes neatly separated. This is the first step to obtain the inner core. The factors correspond to the idealized subjects of our previous terminology. We still have two additional matrices: the principal axes of the situations and the principal axes of the responses. They can serve as linear operators to complete the computation of the inner core and to reduce the situations to idealized situations and the responses to idealized responses. The detailed procedure to obtain the inner core is discussed elsewhere (Levin, 1963; Tucker[4]).

The principal axes matrices of the situations and of the responses were rotated obliquely (Tucker, 1944; Tucker, 1955). The results obtained correspond to the matrices B and C of Equation 10. Due to methodological restrictions, the matrix A corresponding to subject factors could not be computed.

Results

Number of Factors. Three factors were retained for situations and three factors for responses.

The latent roots of the situation matrix are: 3.54, .66, .47, .32, .28, .24, .13, .10, .08, .08, .04. The roots taper off slowly after the third root. The sum of the first three roots is 4.67. The first three roots account for 78% of the variance.

The latent roots of the response matrix are: 3.86, 1.55, .69, .38, .34, .28, .12, .10, .05, .03, .00, $-.03$, $-.05$. The abrupt drop between the third and fourth root, and the slow tapering off from the fourth root onward is here more prominent. The sum of the first three roots is 6.10. The total sum of the roots is 7.56. The first three roots account for 81% of the variance.

The latent roots of ${}_{154}R_{154}$ are: 23.89, 7.39, 6.37, 4.76, 4.28, 4.03, 3.50, 3.24, 2.95, 2.83, etc. There is a sharp drop between the first and second root, but there is also a relatively large decrease from the third to the fourth root. From the fourth root onward there is a slow gradual decline. This can be seen clearly from the differences between each pair of successive roots: 16.50, 1.02, 1.61, .48, .25, .53, .26, .29, .12. The order of magnitude of the first three differences is larger than that of the remaining differences. This points toward the conclusion to retain three factors. Correspondingly, the inner core is, thus, a $3 \times 3 \times 3$ box.

TABLE 6

Factor Loadings of Situations—Oblique Rotation

	I	II	III
1. Auto trip	.01	.04	.38[a]
2. New date	.31[a]	−.12	.30[a]
3. Psychological experiment	.00	.03	.54[a]
4. Ledge high on mountain side	−.06	.56[a]	.01
5. Speech before large group	.55[a]	−.10	.03
6. Counseling Bureau for personal problems	.20	.04	.36[a]
7. Sail boat on rough sea	−.01	.42[a]	.08
8. Competitive contest	.46[a]	.09	−.14
9. Alone in woods at night	.07	.47[a]	−.06
10. Interview for important job	.52[a]	−.03	.04
11. Final exam in important course	.33[a]	.09	.05

[a] Salient loadings.

The Factor Loading Matrices. The rotated factor loadings are given in Tables 6 and 7.

The salient variables of the first factor for situations are:

5. Speech before large group	.55
10. Interview for important job	.52
8. Competitive contest	.46
11. Final exam in important course	.33
2. New date	.31

These are situations where interpersonal stress relations are involved. This factor was called Interpersonal.

TABLE 7

Factor Loadings of Responses—Oblique Rotation

	I	II	III
1. Heart beats faster	.63[a]	.10	−.04
2. Get "uneasy feeling"	.68[a]	−.02	−.05
3. Emotions disrupt actions	.46[a]	−.04	.10
4. Feel exhilarated and thrilled	.11	.60[a]	−.02
5. Want to avoid situation	.34[a]	−.38[a]	.03
6. Perspire	.34[a]	−.03	.18
7. Need to urinate frequently	.07	.09	.49[a]
8. Enjoy the challenge	−.04	.77[a]	.00
9. Mouth gets dry	.21	−.03	.31[a]
10. Become immobilized	.23	−.05	.25
11. Get full feeling in stomach	.05	.05	.42[a]
12. Seek experiences like this	−.05	.73[a]	.05
13. Have loose bowels	−.15	.01	.55[a]
14. Experience nausea	.02	−.09	.41[a]

[a] Salient loadings.

The salient variables of the second factor for situations are:

4. Ledge high on mountain side	.56
9. Alone in woods at night	.47
7. Sail boat on rough sea	.42

These are situations of concern with inanimate personal danger. This factor was called Inanimate.

The salient variables for the third factor for situations are:

3. Psychological experiment	.54
1. Auto trip	.38
6. Counseling Bureau for personal problems	.36
2. New date	.30

The similarity of "Psychological experiment" and "Counseling Bureau" is obvious. A "New date" has a component of similarity to "interpersonal stress." Yet its affinity to the anticipation of a psychological experiment seems cogent. These items may, perhaps, be characterized as comprising situations of "intrapersonal stress." The meaning of "Auto trip" in this context is not clear. For brevity this factor may be labeled Psychological.

The salient variables of the first factor for responses are:

2. Get "uneasy feeling"	.68
1. Heart beats faster	.63
3. Emotions disrupt actions	.46
6. Perspire	.34
5. Want to avoid situation	.34

This factor was called a Distress factor.

The salient variables of the second factor for responses are:

8. Enjoy the challenge	.77
12. Seek experiences like this	.73
4. Feel exhilarated and thrilled	.60
5. Want to avoid situation	−.38

This factor was called an Exhilaration factor.

The salient variables of the third factor for responses are:

13. Have loose bowels	.55
7. Need to urinate frequently	.49
11. Get full feeling in stomach	.42
14. Experience nausea	.41
9. Mouth gets dry	.31

This factor was called Autonomic.

The Core Matrix. The core is a three-mode matrix of order 3 × 3 × 3, that is, it consists of three idealized subjects or types. Each type is an interaction matrix of Situation Factors × Response Factors, that is, for each type we obtain the scores of response factors to the situation factors. The core matrix is given in Table 8. It is arranged in sections corresponding to types. Each section is a 3 × 3 matrix with rows corresponding to response factors and columns to situation factors. Since the items were scored on a scale from "not at all" to "very much"; a low negative entry corresponds to "not at all," a high positive entry corresponds to "very much," and a zero (or near zero) entry corresponds to the average (the scores were converted to standard scores at the first step of the computation).

The first type is high in Distress response (.40, .39, .22) and Autonomic response (.41, .37, .23), and low to low-average in Exhilaration (−.05, −.11, .03). Comparing situations, we find a stronger reaction to the Interpersonal and Inanimate situations than to the Psychological situation.

The second type is notably high in Exhilaration (.29, .44, .32), which is especially conspicuous in the Inanimate situation; but the Distress and Autonomic responses depend on the situation. The anxiousness responses to the Interpersonal factor are somewhere between the Inanimate factor and the Psychological factor: average Distress (−.07) and somewhat high Autonomic (.19). The Exhilaration response (.29) is almost the same as to the Psychological situation (.32).

Comparing situations, we find a very high Exhilaration response (.44) and a low Distress response (−.29) to the Inanimate situation. The Autonomic response is average (.05), but it is not necessarily an anxiousness response—it might be a seasickness type of reaction (note Situations 4 and 7). The response to the Psychological situation is high in all factors: Exhilaration (.32) and the anxiousness factors, that is, Distress and Autonomic (.20, .33). This conforms to the hypothesis in the monograph by Endler et al. (p. 5) that excitement may come both in positive and negative forms.

The third type has high Distress and Autonomic reactions (.30, .37) and low Exhilaration reaction (−.19) to the Inanimate situations, and conversely high Exhilaration (.37, .19) and low to low-average Distress and Autonomic reactions to the Interpersonal and Psychological situations (−.22, −.01, −.16, −.15).

TABLE 8

Core Matrix—Oblique Rotation

	Interpersonal	Inanimate	Psychological
Type I			
Distress	.40	.39	.22
Exhilaration	−.05	−.11	.03
Autonomic	.41	.37	.23
Type II			
Distress	−.07	−.29	.20
Exhilaration	.29	.44	.32
Autonomic	.19	.05	.33
Type III			
Distress	−.22	.30	−.16
Exhilaration	.37	−.19	.19
Autonomic	−.01	.37	−.15

In a schematic and somewhat simplified summary the three types can be described as follows: The first type is noted for general anxiousness reactions: Distress and Autonomic reactions, though he is slightly less sensitive to the Psychological situation. The second type shows a thrilling reaction, especially to physical adventure, and a tendency to get excited to the other situations, that is, to react both with some exhilaration and anxiousness. The third type shows an anxiousness reaction to physical adventure and a thrilling reaction to the other situations.

Conclusion

The main contribution of three-mode factor analysis to classical factor analysis consists of the idea of an inner core and of the method to calculate its numerical values. The emperical studies reported here demonstrate that these values conform to a meaningful pattern. The inner core can be interpreted in terms of psychological types. Each type consists of a matrix which can be explained in terms of interaction among the factors pertaining to the other two modes. The semantic differential study demonstrates a special case consisting of one type only.

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