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DECOMPOSING EVENT-RELATED POTENTIALS: A NEW TOPOGRAPHIC COMPONENTS MODEL *

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"Component" notions inherently used with measurement approaches, raw peak determination, PCA and generator approaches are discussed. By combining aspects of them all, a new model of ERP decomposition is established; quite profitable and surprising mathematical properties are illustrated and discussed.

1. Introduction

Average event-related potentials (ERPs) recorded from the scalp are conceived as composite phenomena having several distinct subprocesses. An important general goal of data analysis is to provide ways to measure these subprocesses and, ideally, to identify them. However, different measurement approaches emphasize different aspects of the data, and, thereby implicit assumptions enter which can be seen as a particular way of defining what a subprocess is. There are three main approaches to the measurement of subprocesses which will be reviewed in the following.

The traditional approach to measuring subprocesses is to determine peaks and latencies (one may subsume area measures as well) of the waveforms in their temporal order. Such a procedure derives from the experience that similar deflections are repeatedly observed for many subjects and experiments. Calling some deflection a "component" requires that for most subjects it exhibits the same polarity within certain latency limits and a similar topographic pattern when studied in comparable experimental situations. Some of them received widely accepted names (e.g., "the N1", "the P3", likewise N100, P300) as they were indeed found within narrow latency ranges and with specific topographic patterns related to particular experimental factors. This

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view to ERPs constitutes a "common sense" and a kind of methodological baseline, and any fresh approach is likely to be measured by its capability to agree with it. Measuring raw peaks has in fact been successful in identifying and pinning down basic properties of event-related potentials. It certainly is a fruitful step towards understanding a set of ERPs, and suffices in many instances to answer what has been asked from the data. Still, component here is a loose and qualitative notion referring to the criteria latency, topography, and reproducibility in a way which defies its use in quantitative modelling.

Traditional peak measurements emphasize the sequential occurrence of the deflections and inherently assume that at peak-time there is just one subprocess active (cf., Donchin & Heffley, 1978). Further problems are subjectivity when determining the peaks, and the difficulty actually obtaining the measures.

As an answer to these issues principal component analysis (PCA) entered the scene and was tutorially popularized by the above paper (see as well, Chapman, McCrary, Bragdon, & Chapman, 1979; Glaser & Ruchkin 1976). It allows that at any latency more than one subprocess contributes and has other profitable features:

- More information of the data is retained: a subprocess is represented by a complete time course and not by just two numbers (amplitude and latency). This also provides a better access to slow processes without a clear peak (but compare Verleger & Möcks, 1987).
- The data of several subjects (sample) is used for quantifying subprocesses, which evidently can strongly improve reliability and determinacy of the measurement. This is a key difference to simple peak measurements in that determining subprocesses becomes directly a statistical approach. This way it is possible to exploit the fact that subprocesses may be present with interindividual stability, being disregarded by approaches proceeding subject by subject.

The PCA approach gained considerable popularity. This is understandable from its advantages but also from the fact that its results were frequently in good accordance with peak measurement findings. It appears that the traditional emphasis on sequentially positioned peaks gave the background for preferring 'simple structure' solutions in PCA as achieved through Varimax rotations – other compelling reasons are seemingly missing. Actual application of PCA and interpreting its results raised a number of issues and there is a continuing debate about its pros and cons, how to apply it properly and whether to use it all (cf., Callaway, Halliday, & Herning 1983; Donchin & Heffley, 1978; see McCallum, Curry, Cooper, Pocock, & Papakostopoulos, 1983; Möcks, 1986; Möcks & Verleger, 1985, 1986; Rockstroh, Elbert, Birbaumer, & Lutzenberger, 1982; Rösler & Manzey, 1981; Verleger & Möcks, 1987; Wastell, 1981; Wood & McCarthy, 1984).

A further distinct attempt to quantify subprocesses aims at localising the electrical generators within the brain. These approaches emphasize the intracranial physiological substrate of the scalp activity; a component belongs to the activity of a fixed piece of brain tissue, commonly modelled by a current dipole. By using biophysical head models the attempt has been made to recompute from scalp potentials the loci of all sources and to obtain the time course of their activity (cf. Nunez, 1981; Scherg & von Cramon, 1985, 1986) or vice versa (e.g., Lutzenberger, Elbert, & Rockstroh, 1987). These approaches proceed subject by subject as simple peak measurement does and interindividual aspects are only studied afterwards.

The notion of components suggested by a generator view as adopted by, for example, Näätänen and Picton (1987) appears well defined by identifying components with scalp contributions of distinct physiological units. Another definite but quite opposite approach was taken by Donchin, Ritter, and McCallum (1978) who stressed the role of experimentally induced variations in determining subprocesses. Both approaches, however, do not explicitly state what is always implicitly assumed: interindividual reproducibility. Speaking of components in either view, of course denotes a reproducible phenomenon which is observable for many or most subjects. It will be shown here that it is quite worthwhile to explicitly refer to interindividual features in decomposition approaches.

In the following section a basic decomposition model is established which appears in accordance to all approaches. By looking more closely at inherent assumptions of PCA and their conceptual limitations, a new model for decomposing ERPs is introduced, which in a way combines aspects of all three approaches. Some important mathematical properties of the new decomposition model are given and discussed, as far as possible, in an intuitive manner. Illustrations with synthetic data follow and some potential concerns with this approach are addressed. Finally the problem of how experimental factors can be specifically incorporated, is discussed.

2. Just one subject

The customary way of applying PCA to ERPs proceeds by using the time points of a fixed interval as "variables" while all other data modes (electrodes × subjects × experimental conditions) enter as "observations". Assume for the moment, that there is just one subject in one experimental condition, such that the data may be arranged in a two dimensional array (a matrix), time × electrodes, which I like to write $x(t, l)$ denoting the data value for time t at electrode l. The approach of PCA then reduces to assuming that there are $k = 1, \ldots, K$ basic functions of time, $c_k(t)$ say, which are common to all electrodes, and there is a set of coefficients $b_k(l)$ independent of time measuring the strength of presence of the component $c_k(t)$ and the l-th electrode. All contributions add up to approximate the data:

$$x(t, l) = \sum_{k=1}^{K} c_k(t) b_k(l). \tag{1}$$

Some remarks: A further symbol for the residual (noise) was, for simplicity, omitted on the right hand side of (1), since the discussion will concentrate on the non-error part of this and following models. The decomposition model (1) is more general than what is usually assumed in PCA, which rather refers to a particular method for solving such a decomposition; here, in slight abuse, PCA stands for this type of model as well.

The decomposition model (1) is in accord with the "generator" approaches. Indeed, if it is assumed, for example, that the activity observed at the electrodes is due to K dipole generators, each having a fixed orientation during the epoch, then model (1) may be straightforwardly derived. (Those are customary conditions on generators used, e.g., by McCarthy & Wood, 1985; Scherg & von Cramon, 1985). Equation (1) is also implied by the more general concept of "aggregates" (Möcks, 1988), which emphasizes the synchrony in time of point current sources and involves distributed generating tissue as well.

Now suppose that there are indeed K dipole generators producing the scalp activity, a situation which can be ideally realized in a simulation (e.g., the data presented by C.C. Wood at the meeting). Do we have a chance to identify their time functions using just the scalp data and relying on the decomposition (1)? Unfortunately not. An infinity of different sets of component functions may be used, with the b's altered appropriately, to reproduce precisely the same scalp data (a familiar statement of this fact is the well known indeterminacy of PCA components with respect to rotations; compare also Möcks & Verleger, 1986; Wood & McCarthy, 1984. Fixing one solution out of the infinity needs more assumptions. The following are made by PCA:

- The coefficients are "uncorrelated" or "orthogonal" which just means to require that $\Sigma_l b_k(l) b_{k'}(l) = 0$ for any two k, k'. If all the true $b_k(l)$ and $b_{k'}(l)$ have the same sign, e.g., if they are due to deep radial dipoles, then this assumption cannot be fulfilled. That is why in PCA usually the mean $x(t, \cdot)$ should be presubtracted producing positive and negative signs for the b's in order to make it at least possible that the assumption is fulfilled.
- At first orthogonal components are required, meaning that $\Sigma_t c_k(t) c_{k'}(t) = 0$ for any k, k', to render the "unrotated" solution. This would be some solution, yet, any other rotated version does as good. Choosing an appropriate rotation is "solved" by convention; customarily the Varimax rotation is used, yielding in general non-orthogonal component functions an output, while preserving orthogonality for the b's (compare Lutzenberger, Elbert, Rockstroh, & Birbaumer, 1981).

These are solely formal mathematical requirements, quite restrictive in fact, in order to ensure that the extraction procedure works, that is ends up with one solution, but they are obviously not driven by subject-matter insights. At best, the Varimax criterion here can be conceived as a hypothesis about the physiological reality, however, entering backdoors in a non-conscious way (Möcks & Verleger, 1986). Valid results in the sense of approximating basal brain processes cannot be guaranteed by PCA, rather one has to put up with a principally descriptive outcome fixed by mathematical stipulation. The generator approaches as well need a set of additional assumptions in order to achieve a definite result from the data, for example Scherg and von Cramon (1985) assume that generators produce at most triphasic waveforms.

Despite these issues model (1) constitutes a biophysically well established basis. It is the point of intersection of PCA and generator approaches. However, the model in its unconstrained form leaves no chance to find its real constituents.

3. Several subjects and a new model

Traditional peak measurements and generator approaches (to date) look at their subprocesses subject by subject. The sample enters only afterwards, when single findings are studied statistically, for example, with regard to homogeneity and scatter across subjects. PCA directly employs the sample when determining its subprocesses; the way this is done can be conceived as an enlargement of (1): The topographical coefficients $b_k(l)$ get a further index i for the subjects yielding $b_k(l, i)$, say, denoting for the ith subject the strength of presence of the k-th component at electrode l. Formally (1) is altered to

$$x(t, l, i) = \sum_{k=1}^{K} c_k(t) b_k(l, i), \tag{2}$$

where, of course, the data x as well has got an additional i. Actual extraction by PCA uses the same constraints as reported above, now applying to the $b_k(l, i)$. Employing (2) implies that:

- Component functions are the same for all subjects.
- Topographical distributions may vary across subjects, and through (2) there is no restriction in whatever manner.

Relating this to the traditional notion of components, emphasis is put on a homogeneous time course across subjects, while homogeneity of topographical patterns stands back. By a same right one could for example stress the latter coming to the following model:

$$x(t, l, i) = \sum_{k=1}^{K} c_k(t, i) b_k(l). \tag{3}$$

Here the i was added to the component functions, allowing that they vary across subjects, while the b's are fixed. A PCA run with model (3) would treat the electrodes as variables and time × subjects as observations. A similar approach was indeed proposed by Skrandies and Lehmann (1982) and was also used in the analysis of spontaneous EEG (Gasser, Möcks, & Bächer, 1983). Giving topography a more prominent role in the decomposition of ERP's is also in the spirit of the various brain-mapping approaches presently mushrooming up (see e.g., Duffy, 1986).

Incorporating the sample via (2) or (3), certainly improves reliability as compared to, for example, running a PCA for each subject separately. Both ways, however, do little to improve the indetermination of the components found; the difficulties remain unchanged as they are a principal property of any unconstrained two-mode decomposition.

In my view, both above approaches for pooling the information of a sample are not convincing. More generally, any attempt to treat the three-mode data $x(t, l, i)$ by some two-mode decomposition is bound to emphasize some aspect while dismissing another. It seems much more suggestive to put time-course on a par with topography, and to use both as defining properties of subprocesses, which would be more sound with the traditional notion of a component yet making it accessable to modelling:

A component in a given epoch is defined by two properties; a fixed time course and a topographic pattern independent of time.

Speaking of a 'component' in this sense, means to speak of two things at the same time; a time-course and a set of topographical coefficients (cf., Gratton, Coles, & Donchin, 1983 and this meeting for a similar view).

It is a key question how the model refers to a sample of subjects. As had been said before, all notions of a component connote a reproducible phenomenon which shows up in a similar manner way for most subjects. In terms of modelling, this aspect represents a kind of structural homogeneity assumption across subjects. The following statement emerges from this assumption:

In each subject the same basic components contribute. Subjects differ by having a specific compound of the components.

In customary PCA, for example, homogeneity constraints enter by using common component functions for all subjects. A component in the present view comprises time-course and topography, and therefore the homogeneity across subjects applies to both features. In other words, taking the two above points together leads to decomposing ERPs through a set of component functions $c_k(t)$ with pertaining electrode coefficient, $b_k(l)$ both common to all subjects, and furthermore a set of scores $a_k(i)$, say, telling the weight of the k-th component in the i-th subject. Written down formally:

$$x(t, l, i) = \sum_{k=1}^{K} c_k(t) b_k(l) a_k(i) \tag{4}$$

Model (4) is also sound with a generator view to components, if it is supplemented by the point that all subjects in an experiment possess common generators. Subject-specific weights of each generator serve to account for interindividual variations in head shape and size, as well as in electrical properties and in functional aspects. Actual generator models (cf., Scherg & von Cramon 1985, 1986) need "unifying" assumptions about interindividual variations when using a "prototype head model" for all subjects.

Model (4) combines some positive features of the different measurement approaches:

- It is sound with the traditional notion of components by giving equal weight to behavior in time and in space (topography) and to interindividual stability.
- It maintains the advantages of PCA and generator approaches over raw peak measurements by using a complete time course for modelling the time behavior of subprocesses.
- It shares with PCA and generator approaches the basic decomposition model (1).
- It goes further than PCA by using a specific way to incorporate interindividual variation.

The notion of components as delineated above refers to structural properties only. Still, these properties suffice to explicitely fix the objective as follows from a crucial mathematical result about model (4), to be discussed in the next section.

4. Properties of the new model

At first sight, model (4) may look more complicated than the one of PCA; instead of two ingredients (loadings and scores), there are three of them now. But as a matter of fact, model (4) is simpler in that it drastically reduces the number of parameters (unknowns) in the model. To give an example, suppose there are $T = 50$ time-points, $L = 5$ electrodes and $N = 20$ subjects; in total 5000 data points. Running a PCA on that with two components, say, would deliver $2 \cdot 50$ values for component functions plus $2 \cdot 5 \cdot 20$ scores, thus a total of 300 numbers which is quite some reduction compared to 5000. However, through model (4), again with two components, there are $2 \cdot 50$ numbers of component functions plus $2 \cdot 5$ topographic coefficients plus $2 \cdot 20$ subject scores, giving a total of 150 numbers only.

It is a general mathematical fact that reducing the number of unknowns while keeping the number of equations is making the remaining parameters more tied or determined. This general insight might help to make plausible, why the drastic reduction of unknowns as provided by model (4) leads to fixing the remaining parameters, meaning that these are uniquely defined:

For a fixed number of components K, the sum appearing on the right hand side of (4) allows one and only one decomposition in terms of the a's, b's, and c's.

The assumptions needed for this to hold are rather weak (in parenthesis a more mathematical formulation is given):

- The K component functions differ in their pattern, and the same holds for the K vectors of subjects scores (the matrices $(c_k(t))$ and $(a_k(i))$ have full rank K).
- The topographical distributions pertaining to any two of the components are not the same (any two of the L-vectors $b_k(l)$ are not collinear).

These assumptions are less restrictive than in PCA. The second one is particularly weak: There may be more components than electrodes without affecting uniqueness; a quite practical asset regarding the set-up of many experiments.

The unique identification of component functions, topographic coefficients, and of subjects scores has the favourable immediate consequence that there is no longer a problem of choosing a rotation, since it is not allowed to apply a rotation to the results; the rotated solution could not reproduce the data. Further, note that there is nothing prescribed about orthogonality or the like. This means on the other hand, that subject scores of two components can correlate with each other and that these correlations are determined as well. As the component functions it means that there may occur almost any overlap in time, without affecting the unique decomposition. Further, topographical coefficients could well possess all the same sign and no subtraction of means or other preprocessing is necessary to approach some requirement. Here, obvious limitations are correlations equal to one in the subject scores, and that component functions or topographical coefficients are identical.

A three-mode model like (4) has been considered before in other frameworks. Here I wish to thank P.C.M. Molenaar, who as a referee brought to my attention the work of Harshman (1970, cited following Harshman & Lundy, 1984a) and Carroll and Chang (1970) and further related papers of which I was unaware when developing model (4) and considering its mathematical properties. The above mentioned articles introduced the same type of model under the names PARAFAC (parallel factors analysis) and CANDECOMP (canonical decomposition), respectively. Kruskal (1976, 1977) studied the uniqueness properties and proved that a set of still weaker conditions than those given here guarantee a unique decomposition. The present conditions, however, keep their significance as they allow a simple mathematical proof (cf., Möcks, 1988) and, more important as they can be shown to ensure that estimates on the basis of model (4) behave "nicely" in a statistical sense

(Pham & Möcks, in preparation). It should be noted that (4) differs from the better known model of three-mode factor analysis (Tucker, 1966) dealing as well with three-mode data. Tucker's model shares non-uniqueness with the customary two-mode models, PCA and factor analysis. A review of the above mentioned work and of further literature was given by Harshman and Lundy (1984a).

The important properties of model (4) follow from its mathematical structure and strongly add to its significance. For many readers being habituated to the notorious indeterminacy of PCA and factor analysis, all these properties might sound too good to be true. This is not the place to present a mathematical proof, but it might be worthwhile trying to make uniqueness a little more plausible (see as well Harshman & Lundy, 1984a). Yet, while model (4) leads in fact to a simpler approach than PCA in terms of data reduction and interpretations, the converse is true for its mathematical treatment. It is quite a difficult problem to make these arguments intuitive, and one should not hope for much clarity or even mathematical precision from the following explanation.

It has been said above that the basic decomposition (model (1)) needs additional constraints to remove its indetermination. To this end, PCA employs further (artificial) restrictions in order to achieve operational uniqueness, and generator approaches use other specific requirements. Model (4) yields uniqueness by the particular way subjects are included in the model: Each subject's data arise from the same basic components (in the new sense, comprising functions and their topographic coefficients). Now suppose, Dr. X, a follower of this model, is running a PCA for each subject separately rendering each time component functions (PCA-loadings) and topographic coefficients (PCA-scores). Since each subject possess a specific compound of the components, each run renders differing component functions and/or differing topographical coefficients, subject by subject. But by hypothesis, these all cannot coincide with the true underlying components, since those ought to be the same for all subjects. Therefore Dr. X tries to find a set of transformations, such that, after having transformed each subject's result, there are identical component functions *and* topographical coefficients. Emphasis is put on the word "and", since the goals were rather straightforwardly achievable addressing just one of both – but (4) requires a same structure for all subjects making Dr. X's task much harder. How many different sets of such subject-transformations can in principle be found by Dr. X? The above assertion says just one, meaning uniqueness of the common solution.

Note that uniqueness not only needs homogeneity across subjects, but also requires that subjects differ – that there is interindividual variation. This dialectic aspect of the unique decomposition is made intuitive by noting, that in an extremely homogeneous sample, where subjects are as like as two peas, considering just one pea suffices, and hence the indeterminacy of the one-subject case returns. Conversely, if there is not enough common structure, parameters can be adjusted at will.

5. Is the model too restrictive? Illustrations

In order to illustrate the issues from a less abstract side, fig. 1 gives synthetic data of five pseudo subjects observed at two pseudo electrodes. On

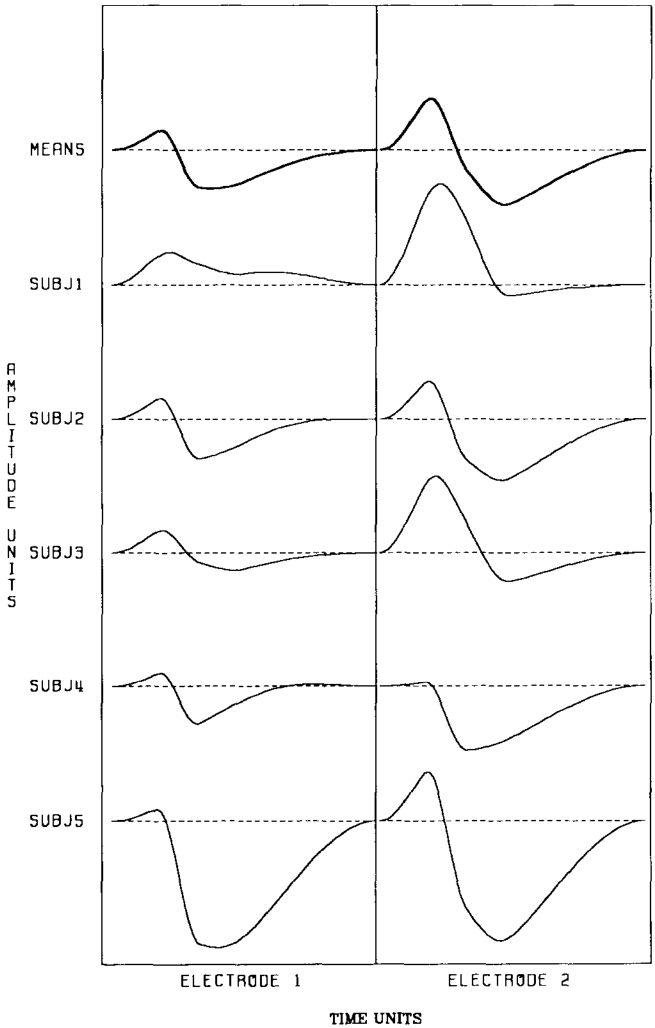

Fig. 1. Synthetic data of five pseudo subjects at two pseudo electrodes. Upper bold traces are grand-means.

top of the figure the grand means are displayed. Starting as a conventional analysis would start, there are two components visible, first a negative one (negative is up, say) followed by a broader positive one. Both components seem more pronounced for the second electrode, at which the second deflection peaks a little later. Apparently there is some latency variation across subjects, in particular for the second deflection. Quite some interindividual variation is visible, the potential of subject 1 at electrode 1 could well contain an artifact. At electrode 2, the potentials of subjects 1 and 4 look quite

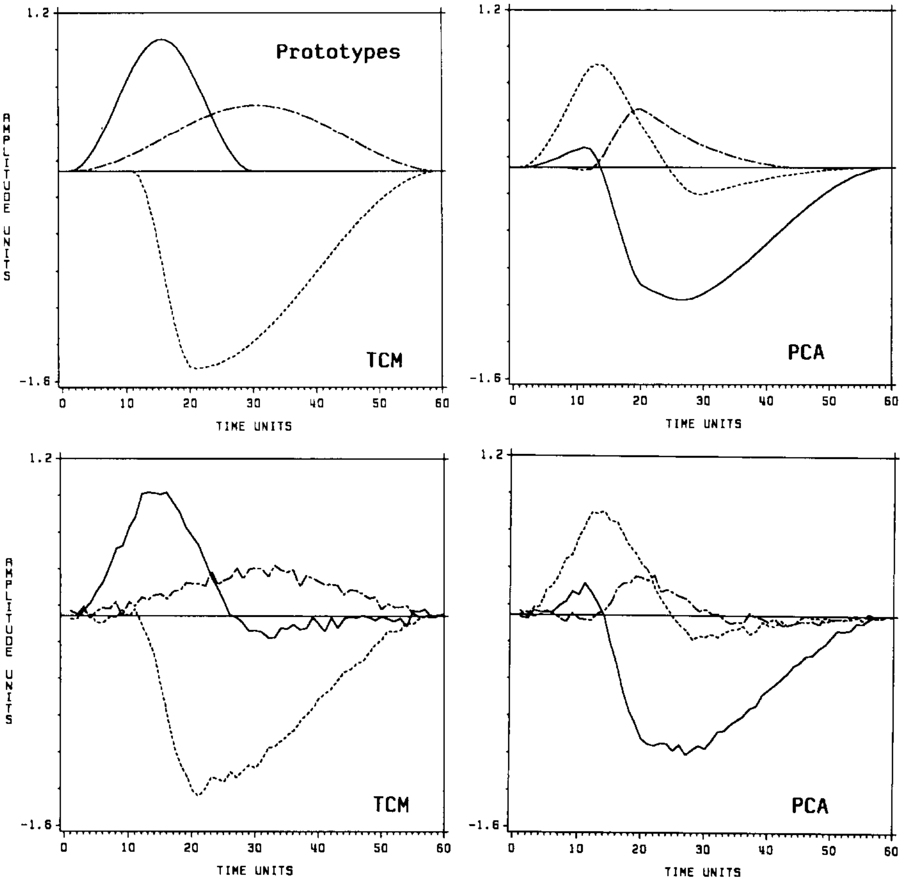

Fig. 2. Left hand side: Protopyes for generating the data of fig. 1, at the same time result of fitting the topographic components model (TCM) in the noiseless case (upper panel) and with additive white noise (lower panel). Right hand side: Result of PCA plus Varimax for the synthetic data in the noiseless case (upper panel) and with noise (lower panel). Components one to three: solid, dashed, and dash-dot.

Table 1
Coefficients for generating the synthetic data set

		Component 1	Component 2	Component 3
Topographic	Electrode 1	3.4	10.0	13.7
coefficients (b's)	Electrode 2	13.7	10.0	3.4
Subject	Subject 1	14.3	2.7	8.0
scores (a's)	Subject 2	5.7	10.2	12.4
	Subject 3	11.6	4.8	4.6
	Subject 4	0.2	9.2	13.5
	Subject 5	8.2	16.7	0.0

distinct.... The list of observations could be continued, peaks and latencies could be determined, and so forth.

However, all the data of fig. 1 were generated by using *three* component functions (prototypes) without noise. These are displayed in fig. 2 (left upper panel). This example demonstrates that model (4) is well capable of explaining a variety of waveshape pattern that may be found in real data. It should be stressed that this is not an extreme case, rather the variety within the reach of the model will substantially increase for more components and electrodes as it is usually the case.

The individual waveforms were generated synthetically based on model (4), and *no latency differences* were introduced. All three prototypes were constructed using the function $\frac{1}{2}(1 + \cos x)$ from $-\pi$ to $+\pi$. Prototypes were numbered according to their peak latency (total interval 60 points). Prototype 1 was defined on 30 points with peak amplitude 1.0, the third one on all 60 points with peak amplitude 0.5. Prototype 2 with onset at point 11, reached its deepest point within 10 points and for the remaining interval slowly returned to zero.

Table 1 gives the topographical coefficients and subject scores of the three prototypes, as chosen for this example. The waveshapes of fig. 1 were then build according to model (4). To be explicit, the data of pseudo subject 1 at electrode 1 was generated by

$$\text{Prototype } 1 \cdot 3.4 \cdot 14.3 + \text{Prototype } 2 \cdot 10.0 \cdot 2.7 + \text{Prototype } 3 \cdot 13.7 \cdot 8.0,$$

and at electrode 2 by

$$\text{Prototype } 1 \cdot 13.7 \cdot 14.3 + \text{Prototype } 2 \cdot 10.0 \cdot 2.7 + \text{Prototype } 3 \cdot 3.4 \cdot 8.0.$$

It might appear surprising that quite some variation of shapes can come out, thought there are for each subject identical component functions and topographic patterns. However, weighted sums of just a few functions with time overlap, will quickly exceed our intuition despite its simple construction. These points should caution against strongly relying on our apperception when analysing ERPs. Approaches focussing on what is "seen" in order to stay "close" to the data might be far off of what really is taking place.

In discussions I heard two main concerns about model (4). The first was phenomenologically oriented asking whether the large interindividual variation of waveshapes mostly encountered in real data is digestible for the model. That may be answered by the example and by the above remarks, and by noting that, of course, a remaining individual specificity can be subsumed in a noise term. The second concern raises the issue of large anatomical variation in cortex shape which in particular could affect the assumption of homogeneous topography, at least for some modalities and experiments. It seems, however, that the subject-specific compound can cope with quite some of this variation still well approximating the homogeneity requirement. To really confound something, the topographical distribution produced by a same generator of an anatomical oddball subject has to be drastically deviant, for example, at least including reversal of signs. This appears not to be a common case and I have never met anyone who hesitated to present grand means per electrode because of possible *anatomical* variations. This objection, of course, would not apply to (4) alone but to other measurement approaches as well. Generator approaches using a unified head model would obviously face a similar problem. Similarly, traditional peak measurements need to assume that in some sense a same peak was determined for all subjects, when performing statistical analysis with the measures. It appears that any measurement approach needs structural homogeneity assumptions in the end, which just happen to be less explicitely stated than in (4). Generally speaking, any modelling and measuring of a phenomenon employs idealizations which cannot and perhaps must not be exactly met in each single case. Idealizations rather open up possibilities to properly account for main features. In my view, the topographic components model (4) constitutes such a set of well-founded and reasonable idealizations.

The flexibility of model (4) can be illustrated by imagining an extreme situation: Suppose there are two subgroups in a sample each having two true underlying components, but quite different ones, that is, a heterogeneous case. Then model (4) still applies by taking four components and by putting to zero the subject scores of the respective set of components. However, it cannot be excluded that model (4) constitutes a poor approximation to the structure of a particular data set. This will be reflected by a poor fit to the raw data despite using many components. More experience with real data will hopefully provide more indicators of such a situation and show how frequent they occur in practice.

The uniqueness property of the model for the data displayed in fig. 1 implies that ten waveshapes of pseudo subjects 1–5 determine uniquely (a) the three prototypes of fig. 2 (left upper panel) and (b) all coefficients compiled in table 1. That is to say, no other set of prototypes and coefficients can be found that would exactly generate these ten curves when using model (4). [1] An algorithm for extracting components according to model (4) and furthermore conventional PCA plus Varimax rotation were applied to this test data set (actually using 30 instead of 5 pseudo subjects). Since the prototypes were exactly identified by the model (4) approach no extra plot is provided for this result; the PCA outcome is displayed in the right upper panel. The lower part shows the results with some white noise added to the data.

It is seen that the topographical component model is still validly approximating the underlying true structure, while PCA in both cases chooses its decomposition as fixed by its formal mathematical constraints (see Möcks & Verleger, 1986; Wood & McCarthy, 1984, for further simulation results with PCA), leading here, for example, to wrong peak latencies of the PCA components as compared to the input prototypes. Unfortunately, I have no access to a generator fitting algorithm, and a comparison with these approaches was not run.

6. Experimental conditions

So far very little has been said about the role of experimental conditions in model (4). In customary PCA experimental conditions [2] are incorporated as a further mode of the observations treated equivalently to different electrodes (see Möcks & Verleger, 1985, for another approach within PCA). From the topographic components model (4) it seems a suggestive and easy way to combine experimental conditions with the subject mode meaning that the index i used before for subjects is to be understood to comprise subjects $\times$ experimental conditions. Doing so implies that experimental factors would act by changing the subject-specific compound of the common component contributions. This, of course, entails the situation that experimental factor evoke a deflection (as e.g., the rare stimulus does in an oddball paradigm) in that zero or almost zero weights for a component in one condition emerges, but high ones in the other. Then subject scores could afterwards be investigated as to the statistical significance of experimental changes.

Another possibility is to consider experimental factors as a further independent mode of the data, that is to say, the four-mode data $x(t, l, i, m)$ where

[1] This is to be understood up to the possibility of rescaling. For example, if all topographic coefficients were multiplier by 17, say, one achieves the same curves by first dividing the subject scores by 17 and so forth.

[2] Here the focus is on "within subject" experimental factors, those realized between groups naturally just add to the subject mode.

m stands for experimental conditions, would be subjected to a four-mode decomposition as follows

$$x(t, l, i, m) = \sum_{k=1}^{K} c_k(t) b_k(l) a_k(i) e_k(m) \tag{5}$$

where $e_k(m)$ gives the weight of the kth component in the experimental condition m. Being independent of subjects, the weights $e_k(m)$ tell the tendency of a component to be present in an experimental condition. Looking at these weights, the component functions and their topographical coefficients provide a concise, still valid way to abstract the data. The subject scores $a_k(i)$ here do not depend on the experimental conditions, and their individual pattern of scores across components could be conceived as a "finger-print" of that particular subject, in that it reflects a property of the subject but not of the experimental conditions, time, or electrodes.

Passing from two-mode to three-mode decompositions gained the uniqueness property, in a similar way a four-mode treatment offers further mathematical advantages over the three-mode case. The assumptions necessary to guarantee uniqueness can be still more relaxed, and a further marked data reduction (reduction in unknowns) is achieved.

7. Conclusion and outlook

The general statement can be made that a closer look at the models of ERP decomposition is quite worthwhile and could lead to substantial methodological progress. The topographic component model presented here appears to be in accord with the traditional way of conceiving ERP data, with statistical aspects of the PCA approach, and with biophysical considerations. The advantages captured are striking, in particular the uniqueness of the decomposition.

The proposed model can be extended towards a four-mode analysis when there are "within subject" experimental factors. It is also possible to generalize it by including individual latency parameters of the component functions, along the lines of Möcks (1986). The approach to individual latencies therein applies to PCA, but can be employed here with much more promise due to the uniqueness property.

The favourable properties of the new model have their costs in terms of complicated mathematical treatment and quite involved problems on the algorithmic side. The algorithmic approach for a direct fit of the model used in PARAFAC and CANDECOMP is indeed an obvious one and, independently, I tried this idea first. At first glance this algorithm should have nice properties; it was found to converge very slow – to an extent which almost prohibits its application in ERP data. On top of this, the algorithm has an unpleasant tendency to produce degenerate solutions (my own experiences are quite in line with those reported by Harshman & Lundy, 1984b). This non-robust behavior made it necessary to develop other approaches, and the results of this paper were obtained with such an alternative algorithm. It turned out to be far quicker and more stable than the convential approach (to be communicated elsewhere). However, still further improved algorithms are strongly desirable in order to make model (4) broadly applicable in ERP analysis.

Apart from algorithmic issues, important mathematical questions remain concerning statistical properties of the estimated coefficients and component functions. Fortunately, some of these questions have already been settled (in cooperation with Pham Dinh Tuan, Grenoble) showing that it will be possible to have access to a broad scope of statistical tools when applying this topographic components model, to list some, significance testing with topographical coefficients, for example, concerning hemispheric differences, confidence bands of component functions and other useful devices.

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